

## Equilibration length of edge states in a GaAs/In<sub>y</sub>Ga<sub>1-y</sub>As/Al<sub>x</sub>Ga<sub>1-x</sub>As quantum well

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We have investigated the nonequilibrium edge-state transport in a pseudomorphic GaAs/In<sub>y</sub>Ga<sub>1-y</sub>As/Al<sub>x</sub>Ga<sub>1-x</sub>As quantum well in the integer quantum-Hall regime. A large deviation of the quantized Hall resistance from the value expected for equilibrium transport is observed when the Landau level filling factor in the bulk region is 4. The dependences of the equilibration length on the number of occupied Landau levels and the variation of the magnetic field within a plateau region are found to be significantly large in comparison with those in GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterostructures. [S0163-1829(97)03104-4]

The resistance of mesoscopic multiterminal structures is described by the Landauer-Büttiker formalism.<sup>1</sup> This model assumes that the device is terminated by perfect reservoirs that supply electrons to all states below the chemical potentials of the reservoirs and absorb all incident electrons without reflection. Although the distinction between the reservoirs and the sample is not clear in real devices, the ambiguity is usually harmless as the electrons experience a large number of inelastic scattering while traveling in terminal leads and equilibrium is readily achieved. The situation, however, changes drastically in the quantum-Hall regime. It was found that nonequilibrium carrier population persists over a macroscopic distance due to the very large equilibration length between edge states.<sup>2</sup> The edge states induced by a magnetic field  $B$  are associated with the Landau levels of a two-dimensional electron gas (2DEG) populated below the Fermi energy. They form quasi-one-dimensional channels with electrons moving along opposite edges of the sample in opposite directions. In a smoothly varying confinement potential, these edge states form compressible strips and have a finite width as a consequence of electron-electron interaction.<sup>3-5</sup> The width of the incompressible strips that separates the compressible strips is typically larger than the extent of the states, which is comparable to the magnetic length  $l_B = \sqrt{\hbar/eB}$ .<sup>4</sup> Therefore, the scattering between the edge states is remarkably suppressed due to the small overlap of the corresponding wave functions for the innermost edge states.<sup>6</sup> If one locally creates a potential barrier by partly depleting the 2DEG underneath a Schottky gate, higher-index edge states are reflected from the barrier while lower-index states remain transparent. The edge channels at the same boundary can be occupied up to different energies and the chemical potential of the reservoir attached to the voltage lead is modified as the transmission of the edge states underneath the gate is varied. Therefore, the Hall resistance is determined as a function of the numbers of transmitted edge states even if the filling factor in the remaining bulk region is unchanged.<sup>7</sup> It has been demonstrated that the nonlocal Hall resistance can be utilized to evaluate the equilibrium length among the edge states.<sup>8</sup>

Recently, we have investigated the magnetic-field dependence of the mixing between edge states in GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterostructures.<sup>9</sup> It was shown that the equilibration length changes in magnitude over a wide range

when the filling factor  $\nu$  is varied within a region where the Hall resistance is quantized.<sup>9</sup> In this paper, we report the equilibration length in a pseudomorphic GaAs/In<sub>y</sub>Ga<sub>1-y</sub>As/Al<sub>x</sub>Ga<sub>1-x</sub>As quantum well. Due to the large potential barrier at the In<sub>y</sub>Ga<sub>1-y</sub>As/Al<sub>x</sub>Ga<sub>1-x</sub>As interface, large carrier densities can be obtained, which in turn lead to a steeper confinement potential near the boundary compared to the well-studied GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As system. A comparison of the equilibration length in different material systems is expected to reveal the characteristics of the edge states.

The quantum well of this study was fabricated by molecular-beam epitaxy. On a semi-insulating (100)GaAs substrate, a 1000-nm-thick undoped buffer layer was grown, followed by an undoped 15-nm In<sub>0.13</sub>Ga<sub>0.87</sub>As quantum well. After the deposition of 4 ML GaAs at the In<sub>0.13</sub>Ga<sub>0.87</sub>As growth temperature, the segregated In was reevaporated by a rapid heating of the substrate during a growth interruption.<sup>10</sup> The upper cladding layer consists of an undoped 5-nm Al<sub>0.3</sub>Ga<sub>0.7</sub>As spacer layer, a Si  $\delta$ -doped layer with the doping concentration of  $2.5 \times 10^{16} \text{ m}^{-2}$  embedded in the middle of a 6-ML-thick undoped GaAs layer, a 94-nm undoped Al<sub>0.3</sub>Ga<sub>0.7</sub>As layer, and a 20-nm undoped GaAs cap layer. The substrate temperature was 510, 550, and 580 °C for the growth of In<sub>0.13</sub>Ga<sub>0.87</sub>As, GaAs, and Al<sub>0.3</sub>Ga<sub>0.7</sub>As, respectively.

The electron density and the mobility at the temperature  $T=0.3$  K in the dark were  $1.2 \times 10^{16} \text{ m}^{-2}$  and  $5.1 \text{ m}^2/\text{V s}$ , respectively. The corresponding mean free path is  $l_0=0.71 \mu\text{m}$ . Despite the high carrier density in the sample, only the lowest two-dimensional subband is occupied as evidenced by the Shubnikov-de Haas oscillation shown in Fig. 1. This agrees well with the result of a self-consistent calculation.<sup>11</sup> The samples were patterned to a Hall bar geometry by conventional photolithography. We note that the devices were defined by a deep etch, and so the quantum well is exposed to air at the side wall. The lower-left inset of Fig. 2 illustrates the geometry of the samples.<sup>9</sup> A hole was etched in the middle of the Hall bar. After alloyed AuGe/Ni contacts were processed (hatched squares), the Schottky gates (shaded areas) were evaporated across each of the two parallel channels to modify the density in the 2DEG channels. The distance between the gates along the edge of the hole is  $L=20 \mu\text{m}$ . The resistance was measured using standard lock-in technique with a constant current of  $\leq 10 \text{ nA}$  at  $T=0.3 \text{ K}$ .

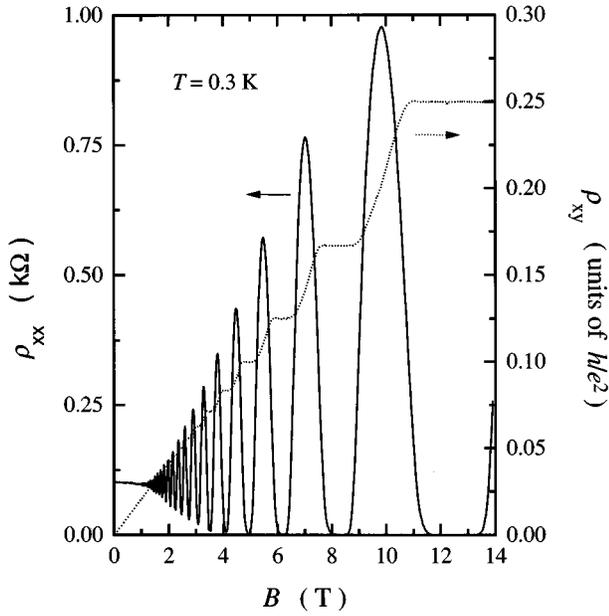


FIG. 1. Shubnikov–de Haas oscillation and quantized Hall effect in unpatterned sample. Despite the large carrier density only the lowest subband is populated.

A typical edge-state configuration in the device is shown in the upper-right inset of Fig. 2. In a high magnetic field, the edge states move along the outer boundary in a clockwise manner, whereas those around the hole circulate with a counterclockwise rotation. If a negative voltage is applied to the gate, the 2DEG under the gate is partly depleted. Consider a situation where the lower-lying  $K_i$  states are transmitted under the gate, while the rest of the  $N - K_i$  states are reflected. Here  $N$  is the number of occupied Landau levels in the bulk region. The incoming states on either side of the hole are originally filled up to the chemical potentials of the reservoirs 1 or 3. If these edge states exchange electrons via the circulating states, the chemical potentials of the outgoing states can take values between those of the reservoirs of the current contact. Since this mixing leads to a quantization of the Hall resistance at arbitrary values, the quantized value can be employed to estimate the extent of equilibration.<sup>9</sup> In the adiabatic limit, where no mixing takes place while the edge states travel between the gates, the generalized Hall resistance  $R_H = R_{13,24}$ , which indicates that the current flows from lead 1 to lead 3 and the voltage difference is measured between leads 2 and 4, is given as

$$R_H^{\text{ad}} = \frac{h}{e^2} \frac{1}{K}, \quad (1)$$

where  $K = \max(K_1, K_2)$ . In the equilibrium limit, however, the current redistributes in the left- and right-hand side of the hole. Consequently, an equilibrium population is realized. In this case,  $R_H$  is directly determined by  $K_1$  and  $K_2$ :

$$R_H^{\text{eq}} = \frac{h}{e^2} \frac{N^2 - K_1 K_2}{N[(K_1 + K_2) - 2K_1 K_2]}. \quad (2)$$

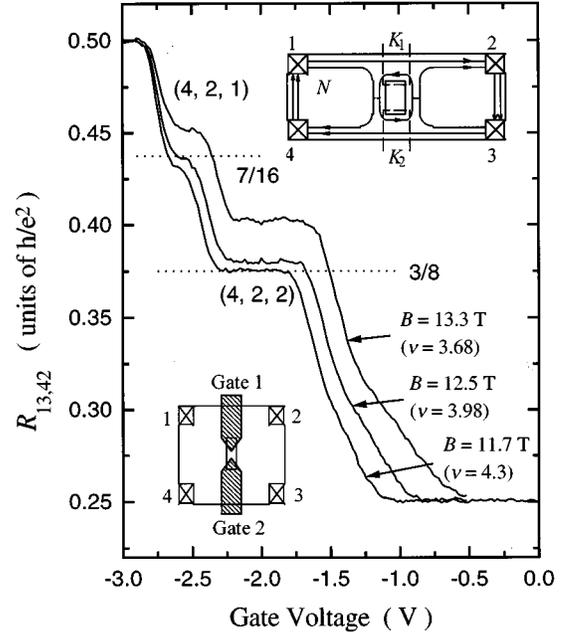


FIG. 2. Hall resistance at  $T=0.3$  K as a function of the bias voltage applied to gate 2 when  $N=4$  in the bulk at different magnetic fields  $B$ . The bias voltage to gate 1 is fixed such that the number of transmitted edge states is 2. The dotted lines indicate the plateau value in the equilibrium limit. In the adiabatic limit  $R_{13,42} = \frac{1}{2}e^2/h$ . The lower-left inset shows a schematic layout of the structure with the contacts being labeled 1–4. Typical edge-state configuration at the Fermi energy is shown in the upper-right inset. The numbers of occupied edge states are  $N$  in the bulk and  $K_1$  and  $K_2$  under gates 1 and 2, respectively. Note that  $K_1 = K_2$  in the figure.

To determine the gate voltage dependence of the filling factor underneath both gates, we separately pinched-off one of the channels by a strong negative gate voltage ( $\sim -3.5$  V). The magnetic field was adjusted such that the number of occupied Landau levels in the bulk is  $N$ . By sweeping the gate voltage to the other channel,  $R_H$  is quantized at  $h/Ke^2$  when an integer filling factor  $K$  is achieved under the gate. We have measured  $R_H$  for various sets of  $N$ ,  $K_1$ , and  $K_2$ .

Figure 2 shows  $R_H$  with slightly different filling factors  $\nu$  in the bulk for  $N=4$  and  $K_1=2$ . When the channel below gate 2 is completely depleted,  $R_H$  is quantized exactly at  $h/2e^2$  as expected from the filling factor below gate 1. On the other hand,  $R_H = h/4e^2$  when the gate voltage is zero. It is, thus, indicated that the edge states are well established within this magnetic-field range. For the combination of the filling factors  $(N, K_1, K_2) = (4, 2, 2)$ , the plateau value in  $R_H$  deviates from that in the equilibrium limit (indicated by the dotted line in Fig. 2). The deviation increases with increasing  $B$ . A similar deviation from the equilibrium limit is also found for  $(4, 2, 1)$ . In the latter case, however, the spin splitting is not completely resolved, and so the plateau value when  $B = 11.7$  T was slightly below the lower bound of the theoretical prediction. Note that we find no plateaulike feature in the gate-voltage dependence corresponding to  $(4, 2, 3)$ . Therefore, we restrict our attention to the spin-degenerate case.

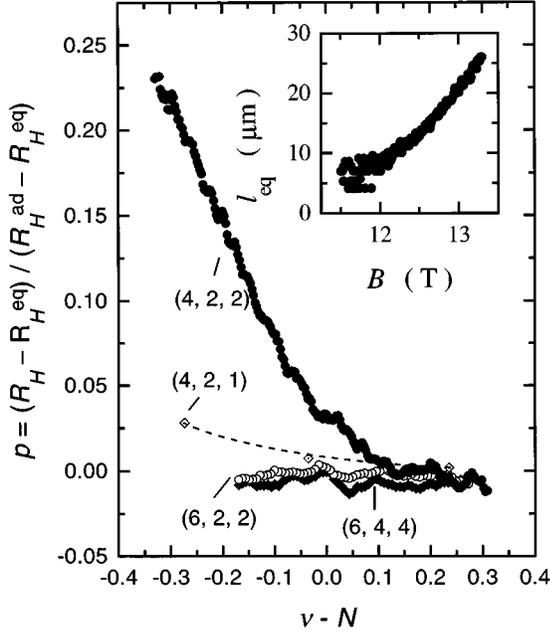


FIG. 3. Plateau value of the Hall resistances normalized by that in the adiabatic and equilibrium limits as a function of the deviation of the filling factor  $\nu$  in the bulk from the integer value  $N$ . Data shown are taken in the magnetic-field range where the Hall resistance is quantized. The line for (4,2,1) is a guide to the eye. Inset: Equilibration length  $l_{\text{eq}}$  for (4,2,2) as a function of the magnetic field.

We have plotted in Fig. 3 the plateau value normalized by that in the adiabatic and equilibrium limits,  $p = (R_H - R_H^{\text{eq}}) / (R_H^{\text{ad}} - R_H^{\text{eq}})$ , as a function of the deviation of the filling factor  $\nu$  in the bulk from the integer value  $N$  for several combinations of  $(N, K_1, K_2)$ . We expect  $p = 0$  and 1 for fully equilibrium and adiabatic transport, respectively. When only two edge states are relevant to the transport,  $p$  is regarded to represent the amount of nonequilibrium electrons. For  $N = 6$  the edge states are completely equilibrated in all cases we examined. On the other hand, for  $N = 4$  at the maximum nearly 25% of the electrons remain unscattered for (4,2,2) at higher  $B$ . In the case of the mixing between two edge states,  $p$  is related to the equilibration length  $l_{\text{eq}}$  as<sup>8,9</sup>

$$p = \exp(-2L/l_{\text{eq}}). \quad (3)$$

The equilibration length for (4,2,2) is estimated as shown in the inset of Fig. 3. The maximum value of  $l_{\text{eq}}$  observed in the device is 25  $\mu\text{m}$ . This is comparable to the value we previously found in devices patterned from GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterostructures although  $l_0$  in the GaAs/In<sub>y</sub>Ga<sub>1-y</sub>As/Al<sub>x</sub>Ga<sub>1-x</sub>As quantum well is about an order of magnitude smaller than that in the GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterostructures.

The scattering rate between adjacent edge states is primarily determined by the overlap of the wave functions, and so it depends strongly on the width of the incompressible strip that separates the edge states.<sup>3</sup> Chklovskii, Shklovskii, and Glazman<sup>4</sup> estimated the width  $a_{N-1}$  of the  $(N-1)$ th incompressible strip which separates the  $N$ th and the  $(N-1)$ th edge channels as

$$\frac{a_{N-1}}{l_B} \propto \sqrt{a_B} \frac{1}{\nu - N + 1}, \quad (4)$$

where  $a_B = 4\pi\epsilon\hbar^2/me^2$  is the effective Bohr radius. One can roughly expect that the scattering rate is proportional to  $\exp(-a_{N-1}/l_B)$ . Equation (4) indicates that the scattering rate between the edge states is a function of solely  $\nu - N$ . One hence expects comparable equilibration lengths for a different number of occupied Landau levels. Experimentally, however, a shorter equilibration length is found when  $N$  is increases.<sup>9,12</sup> We emphasize that the dependence of  $l_{\text{eq}}$  on  $N$  or  $\nu - N$  is remarkably large in the GaAs/In<sub>y</sub>Ga<sub>1-y</sub>As/Al<sub>x</sub>Ga<sub>1-x</sub>As quantum well in comparison to that in similar patterned GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterostructures. On the other hand,  $l_{\text{eq}}$  decreased from 36 to 15  $\mu\text{m}$  when  $\nu - N$  is changed from 0.2 to  $-0.3$  in Ref. 9. Furthermore,  $l_{\text{eq}}$  decreased from 25 to 3  $\mu\text{m}$  in the GaAs/In<sub>y</sub>Ga<sub>1-y</sub>As/Al<sub>x</sub>Ga<sub>1-x</sub>As quantum well. Additionally, we find that the transport is completely equilibrated for  $N = 6$  over the distance of 20  $\mu\text{m}$  in the GaAs/In<sub>y</sub>Ga<sub>1-y</sub>As/Al<sub>x</sub>Ga<sub>1-x</sub>As quantum well, whereas  $l_{\text{eq}}$  is suppressed only by roughly 50% between  $N = 4$  and 6 in the GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterostructures.

The different behavior of  $l_{\text{eq}}$  is expected to be originating from the difference of the material parameters and the carrier density. The larger  $a_B$  in the GaAs/In<sub>y</sub>Ga<sub>1-y</sub>As/Al<sub>x</sub>Ga<sub>1-x</sub>As quantum well will result in a smaller scattering rate between the edge states, in accordance with the experimental observation. The screening of the potential is effective as the carrier density is large, and so the potential profile near the boundary of the sample is considered to be more hard-well-like in the case of the GaAs/In<sub>y</sub>Ga<sub>1-y</sub>As/Al<sub>x</sub>Ga<sub>1-x</sub>As quantum well. Consequently, the formation of the strips will be less pronounced in the vicinity of the side wall. This may account for the rapid suppression of the nonequilibrium transport when the edge states are pushed towards the boundary of the sample by increasing  $\nu$  larger than the integer value. The same value of  $N$  is achieved at higher  $B$  for higher carrier densities. Therefore, the broadening of the Landau levels due to the low mobility may be less important as the separation between the levels is large.

In conclusion, we have shown that, despite the short mean free path, nonequilibrium edge-state transport can be maintained as long as  $\sim 25$   $\mu\text{m}$  in the GaAs/In<sub>y</sub>Ga<sub>1-y</sub>As/Al<sub>x</sub>Ga<sub>1-x</sub>As quantum well. This value is comparable to that found in GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterostructures with identical sample geometry and much longer mean free path. The equilibration length is found to be strongly dependent on  $N$  and  $\nu - N$  in the GaAs/In<sub>y</sub>Ga<sub>1-y</sub>As/Al<sub>x</sub>Ga<sub>1-x</sub>As quantum well. These different characteristics can be explained in terms of the different material parameters and the large carrier density in the GaAs/In<sub>y</sub>Ga<sub>1-y</sub>As/Al<sub>x</sub>Ga<sub>1-x</sub>As quantum well.

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<sup>1</sup>See, for instance, M. Büttiker, in *Semiconductors and Semimetals*, edited by M. Reed (Academic, New York, 1992), Vol. 35, p. 191.

<sup>2</sup>See, for instance, R. J. Haug, *Semicond. Sci. Technol.* **8**, 131 (1993), and references therein.

<sup>3</sup>C. W. J. Beenakker, *Phys. Rev. Lett.* **64**, 216 (1990); A. M. Chang, *Solid State Commun.* **74**, 871 (1990).

<sup>4</sup>D. B. Chklovskii, B. I. Shklovskii, and L. I. Glazman, *Phys. Rev. B* **46**, 4026 (1992).

<sup>5</sup>A. V. Khaetskii, V. I. Fal'ko, and G. E. W. Bauer, *Phys. Rev. B* **50**, 4571 (1994).

<sup>6</sup>B. W. Alphenaar, P. L. McEuen, R. G. Wheeler, and R. N. Sacks,

*Phys. Rev. Lett.* **64**, 677 (1990).

<sup>7</sup>B. J. van Wees, E. M. M. Willems, C. J. P. Harmans, C. W. J. Beenakker, H. van Houton, J. G. Williamson, C. T. Foxon, and J. J. Harris, *Phys. Rev. Lett.* **62**, 1181 (1989).

<sup>8</sup>G. Mueller, D. Weiss, A. V. Khaetskii, K. von Klitzing, S. Koch, H. Nickel, W. Schlapp, and R. Loesch, *Phys. Rev. B* **45**, 3932 (1992).

<sup>9</sup>Y. Takagaki, K.-J. Friedland, J. Herfort, H. Kostial, and K. Ploog, *Phys. Rev. B* **50**, 4456 (1994).

<sup>10</sup>O. Brandt, K. Ploog, L. Tapfer, M. Hohenstein, and F. Phillip, *J. Cryst. Growth* **115**, 99 (1993).

<sup>11</sup>J. Herfort and K.-J. Friedland (unpublished).

<sup>12</sup>S. Komiyama and H. Nii, *Physica B* **184**, 7 (1993).