Space-charge waves in semiconductors excited by static and moving optical interference patterns

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We consider space-charge waves in semiconductors that are excited by the superposition of optically generated static and moving interference patterns. Induced dc and ac contributions are resonantly enhanced when the spatial and temporal periods of the interference grating match the related quantities of the space-charge wave. Two eigenmodes of the electron-hole system are identified. One of them is excited by the synchronous drift of photogenerated electron-hole pairs with the moving intensity grating. The other one has the character of trap-recharging waves. The amplification of induced dc and ac components provides complementary information about space-charge waves in semiconductors. © 2005 American Institute of Physics.

I. INTRODUCTION

A class of experiments mainly for materials characterization has been implemented, especially for amorphous films and later also for semiconductors that rely on the spatial and temporal evolutions of photoexcited carriers. In early studies, a sinusoidal light intensity pattern with a spatial period was created by two coherent laser beams to determine the ambipolar diffusion length in semiconductors. In a next step, in order to simultaneously measure the lifetime and ambipolar mobilities, the temporal information provided by a moving interference grating was additionally taken into account. The moving interference pattern is produced by two laser beams of different intensities and slightly different frequencies, which interfere and create an intensity grating on the sample surface that moves with a constant velocity \( v_{gr} \). The photogenerated space charges and the electric field accompanying them give rise to a dc short-circuit current, which is the quantity measured in the experiment as a function of the velocity \( v_{gr} \). This so-called moving-photocarrier-grating technique has been successfully applied to measure the bipolar transport parameters of photoexcited materials. Both techniques using either a static or a moving interference pattern to determine the materials parameters do not rely on the application of an external dc bias. However, a number of interesting physical phenomena can be studied, when in addition an electric field is applied. In the illuminated biased sample, the photogenerated electrons and holes move in opposite directions with their respective drift velocities \( v_n \) and \( v_p \). This drift gives rise to a resonance of the modulation field amplitude, when the bright interference stripes of the moving pattern travel synchronously with the charge cluster that it generates. For electrons, the resonance condition is expressed by \( v_{gr} = v_n \). Based on the synchronous drift of photogenerated carriers with the moving interference pattern, Dolfi et al.

II. BASIC EQUATIONS

Let us treat two sets of two coherent laser beams that are brought to interfere on the surface of a semiconducting sample. The first set of beams creates an intensity grating (cf. Fig. 1) with the spatial period \( \Lambda = \lambda / [2 \sin(\alpha/2)] \), where \( \alpha \) is the angle between the two beams and \( \lambda \) the laser wavelength. For the second set of two beams, a small frequency shift \( \Delta f \) between them is realized by an acousto-optic modulator. The interference pattern produced by this configuration moves with a velocity \( v_{gr} = \Delta f / \lambda \) in the \( x \) direction. In the one-
is expressed by $y$ grating has the wave vector $K_g$.

The electro- and hole distributions $g_s$ and $g_m$ semiconductor, where the carrier concentration in the dark is

dimensional approximation, the static and moving interference patterns lead to a generation rate for electrons and holes given by

$$g(x,t) = g_0 + g_s \cos(K_g x) + g_m \cos(K_g x - \Omega t),$$

which consists of a homogeneous part $g_0$ and two modulated contributions. $g_s$ and $g_m$ denote the generation rates of the static and moving interference patterns, respectively. The grating has the wave vector $K_g = 2\pi/\Lambda$, and the frequency $\Omega$ is expressed by $v_g$ via $\Omega = K_g v_g$. We consider an intrinsic semiconductor, where the carrier concentration in the dark is negligible compared to the photocarrier densities, and focus on the small modulation regime, when the intensities of each pair of laser beams remarkably differ from each other so that $g_s$ and $g_m \ll g_0$. The modulated generation rate $g(x,t)$ induces electron and hole distributions $n(x,t)$ and $p(x,t)$ that exhibit the same spatial and temporal periodicities as $g(x,t)$. The dynamics of photogenerated electrons and holes is treated by continuity equations, which take into account the inhomogeneous generation rate as well as the diffusion, drift, and recombination of carriers that accompany the generation. The continuity equations

$$\frac{\partial n(x,t)}{\partial t} = \frac{1}{e} \frac{\partial J_n(x,t)}{\partial x} + g(x,t) - r(x,t),$$

$$\frac{\partial p(x,t)}{\partial t} = -\frac{1}{e} \frac{\partial J_p(x,t)}{\partial x} + g(x,t) - r(x,t),$$

encompass carrier generation $g(x,t)$ and recombination $r(x,t)$ rates. The current densities for electrons [$J_n(x,t)$] and holes [$J_p(x,t)$] result from the respective drift and diffusion contributions

$$J_n(x,t) = e n(x,t) \mu_n E(x,t) + e D_n \frac{\partial n(x,t)}{\partial x},$$

$$J_p(x,t) = e p(x,t) \mu_p E(x,t) - e D_p \frac{\partial p(x,t)}{\partial x},$$

with $\mu_n$ and $\mu_p$ ($D_n$ and $D_p$) denoting the mobilities (diffusion coefficients) for electrons and hole, respectively. The total electric field $E(x,t)$ is the sum of the constant electric field $E_0$ applied along the $x$ direction and the space-charge field $\delta E(x,t)$, which is due to locally unbalanced electron and hole densities as described by the Poisson equation

$$\frac{\partial \delta E(x,t)}{\partial x} = \frac{4\pi e}{\varepsilon} [p(x,t) - n(x,t)],$$

where $\varepsilon$ denotes the dielectric constant. The constant electric field $E_0$ provided by the external voltage $U$ acts on the photogenerated electrons and holes so that they move in opposite directions with their drift velocities $v_n$ and $v_p$. This carrier motion, which is influenced by the intensity grating, gives rise to a current in the outer circuit, in which a load resistor $R_L$ is placed. This current is determined by Kirchhoff’s law, which gives us the boundary condition to solve the set of differential equations for the density fluctuations $\delta n$ and $\delta p$ and the space-charge field $\delta E$. A moving grating alone induces only a constant current. The generation of a time-dependent, alternating current requires the additional illumination of the sample so that a static grating is provided. Kirchhoff’s law has the form

$$E_0 + \frac{1}{L} \int_0^L dx \delta E(x,t) + \rho I(t) = \frac{U_0}{L},$$

where $\rho = R_L S/L$. $L$ and $S$ denote the length and cross section of the sample, respectively.

The coupled differential Eqs. (2)–(6) are treated in the limit of weak modulation, where small density fluctuations $\delta n(x,t)$ and $\delta p(x,t)$ around the uniform carrier densities $n_0$ and $p_0$ are considered ($n = n_0 + \delta n$, $p = p_0 + \delta p$). Due to the global charge neutrality under homogeneous illumination, we have $n_0 = p_0 = g_0$, where $\tau$ denotes the recombination lifetime. In the small signal approximation, the recombination rate is given by

$$r(x,t) = g_0 + \frac{1}{2\tau} (\delta n + \delta p).$$

We are going to reformulate Eqs. (2)–(5) for the variations of the carrier densities from their constant mean values in terms of the quantities

$$f_\pm(x,t) = n(x,t) \pm p(x,t).$$

Similar short-hand notations are used for the mobilities ($\mu_\pm$), diffusion coefficients ($D_\pm$), and current densities ($J_\pm$). Taking into account the decomposition of $n(x,t)$ and $p(x,t)$ into a homogeneous and modulated part, we obtain

$$f_+(x,t) = f_0^+ + \delta f_+(x,t)$$

and

$$f_-(x,t) = \delta f_-(x,t),$$

and a similar relation for the current $I(t) = I_0 + \partial I(t)$. The constants $f_0^+$ and $I_0$ denote the total carrier density and total
current, respectively, under uniform illumination \((g_x = g_m = 0)\). These quantities are calculated from

\[ f^0_+ = 2g_0e\tau, \quad I_0 = \sigma_\mu E_0, \quad \sigma_\mu = e\mu_+ g_0\tau, \]

(11)

Together with the equation for the electric field \(E_0\)

\[ (1 + \rho \sigma_\mu)E_0 = \frac{U_0}{L}. \]

(12)

To calculate the modulated quantities \(\delta E, \delta I, \delta f_+, \) and \(\delta f_-\), it is convenient to introduce dimensionless independent variables

\[ \tilde{x} = K_x x, \quad \tilde{t} = \Omega t, \]

(13)

dimensionless functions

\[ Y(\tilde{x}, \tilde{t}) = \frac{\delta E(\tilde{x}, \tilde{t})}{E_0}, \quad f(\tilde{t}) = \frac{\delta I(\tilde{t})}{I_0}, \]

(14)

\[ \lambda_\mu(\tilde{x}, \tilde{t}) = \frac{\delta f_+(\tilde{x}, \tilde{t})}{f^0_+}, \]

(15)

and dimensionless parameters

\[ \Lambda_\mu = D_4 K_x/(\mu_+ E_0), \]

(16)

\[ \mu = \mu_- \mu_+, \quad \kappa = (e/4\pi\varepsilon)E_0 K_x f^0_+. \]

(17)

Besides the scattering time \(\tau\), the approach depends on a second characteristic time, namely, the Maxwellian relaxation time \(\tau_M = \varepsilon\tau/(4\pi\sigma_\mu)\), which is sometimes referred to as the dielectric relaxation time. Taking into account the Poisson equation [Eq. (6)], the dimensionless form of the continuity Eqs. (2) and (3) is given by

\[ \Omega \tau \frac{\partial \lambda_\mu}{\partial \tilde{t}} + \lambda_\mu = h(\tilde{x}, \tilde{t}) + d_\mu \frac{\partial}{\partial \tilde{x}} \left\{ \mu Y + (\lambda_- + \mu \lambda_\mu)(1 + Y) \right. \]

\[ \left. + \Lambda_\mu \frac{\partial \lambda_\mu}{\partial \tilde{x}} + \Lambda_- \frac{\partial \lambda_-}{\partial \tilde{x}} \right\}, \]

(18)

\[ \frac{\partial \lambda_-}{\partial \tilde{t}} = -\kappa \frac{\partial^2 Y}{\partial \tilde{x}^2} \frac{\partial \lambda_-}{\partial \tilde{t}}, \]

(19)

with the modulated generation rate

\[ h(\tilde{x}, \tilde{t}) = \frac{g_x}{g_0} \cos(\tilde{x}) + \frac{g_m}{g_0} \cos(\tilde{x} - \tilde{t}), \]

(20)

and the parameter \(d_\mu = \mu_+ E_0 K_x \tau/2\). In the presence of a moving grating, but in the absence of the static interference pattern, the solutions of Eqs. (18)–(20) depend only on the difference \(\tilde{x} - \tilde{t}\) and not on the spatial parameter \(\tilde{x}\) and the time variable \(\tilde{t}\) independently. This dependence is in accordance with Eq. (20) under the condition \(g_x = 0\). In this case, the partial differential Eqs. (18) and (19) reduce to a set of ordinary differential equations, in which the explicit \(\tilde{t}\) dependence disappears. As a consequence, only a constant current is induced in the outer circuit by the moving intensity grating. To induce also an alternating current, an additional static interference pattern is necessary.

We proceed by integrating the equation for \(f_-(x,t)\) over the \(x\) coordinate in order to derive the required third equation. In this equation, an integration constant appears, which is given by the total time-dependent current \(I(t)\). We obtain the result

\[ f(\tilde{t}) = \Omega \tau \frac{\partial Y}{\partial \tilde{t}} + Y + (\lambda_\mu + \mu \lambda_-)(1 + Y) + \lambda_- \frac{\partial \lambda_-}{\partial \tilde{x}}, \]

(21)

which is simultaneously solved with the integral Eq. (7). The solution of these nonlinear partial differential equations is simplified by the observation that the solution exhibits the same periodicity as the source term \(h(\tilde{x}, \tilde{t})\). This symmetry is accounted for by a discrete Fourier representation for the induced relative carrier densities \(\lambda_\mu\) and space-charge function \(Y\)

\[ Y(\tilde{x}, \tilde{t}) = \sum_{p, l = 0}^\infty e^{i(p\tilde{x} + il\tilde{t})} Y_{p,l}. \]

(22)

The Fourier coefficients of the source term \(h(\tilde{x}, \tilde{t})\) are given by

\[ h_{p,l} = \frac{1}{2g_0} [g_x \delta_{0,0}(\delta_{p,1} + \delta_{p,-1}) + g_m (\delta_{p,1} \delta_{l,-1} + \delta_{p,-1} \delta_{l,1})] \]

(23)

\[ = h_{p,-l}. \]

Performing the discrete Fourier transformation, the coupled partial differential Eqs. (18)–(21) and the boundary condition (7) are reformulated in terms of the following set of coupled nonlinear equations for the Fourier coefficients:

\[ Y_{0,l} = -\rho \sigma_\mu f_+, \quad \lambda_{-p,l} = -i p \kappa Y_{p,l}, \]

(24)

\[ f_0 \delta_{p,0} = [1 + i \Omega \tau_M - ip \kappa \mu + p^2 \kappa^2 \lambda_-] Y_{p,l} + (1 + i p \Lambda_-) \lambda_{+p,l} \]

\[ + \sum_{p', l'} [\lambda_{+p-p', l'-l'} - i(p - p') \kappa \mu Y_{p-p', l'}] Y_{p', l'}, \]

(25)

\[ [1 + i \Omega \tau - ip d_\mu + p^2 d_\mu \Lambda_\mu] \lambda_{+p,l} \]

\[ = h_{p,l} + ip d_\mu [\mu - i p \kappa + p^2 \kappa^2 \mu \lambda_-] Y_{p,l} \]

\[ + ip d_\mu \sum_{p', l'} [\mu \lambda_{+p-p', l'-l'} - i(p - p') \kappa Y_{p-p', l'}] Y_{p', l'}. \]

(26)

Proceeding in the same way as in Refs. 14 and 15, an analytical solution of these exact equations can be derived within the weak modulation limit \((g_x \ll g_m \ll 0)\) from recurrence relations for the Fourier coefficients \(Y_{p,l}\) and \(\lambda_{p,l}\). Note that the description of SCWs and related amplification effects require the treatment of nonlinear contributions of the field modulation.14,15 Within the linear approximation as adopted by Hundhausen et al.9 the excitation of SCWs as well as related current resonances cannot be properly described. In our perturbational solution of Eqs. (24)–(26), we treat Eq. (25) for \(p = 0\) and retain its nonlinear contributions on the right-hand side. To find the lowest-order Fourier coefficients \(f_0\) and \(f_{\pm 1}\), we need the quantities \(Y_{\pm 1,l}\) and \(\lambda_{\pm 1,l}\).
calculated to lowest order with respect to the small parameters $g_{m}/g_{0}$ and $g_{l}/g_{0}$. The result

$$-N_{p,l}y_{p,l} = h_{p,l}(1 + ip\Lambda_{-}).$$

(27)

$$\lambda_{p,l} = -\frac{y_{p,l}}{(1 + ip\Lambda_{-})}[1 + i\Omega\tau_{M} - ip\kappa \mu + p^2\kappa \Lambda_{+}],$$

(28)

together with the short-hand notation

$$N_{p,l} = [1 + i\Omega\tau_{M} - ip\kappa \mu + p^2\kappa \Lambda_{+}][1 + i\Omega\tau - ipd_{s} \mu + p^2d_{s} \Lambda_{+}] + ipd_{s}(1 + ip\Lambda_{-})[\mu - ip\kappa + p^2\kappa \Lambda_{-}],$$

(29)

is used for the calculation of the induced constant and alternating current contributions in Secs. III and IV.

### III. INDUCED CONSTANT CURRENT

The application of a constant electric field $E_{0}$ to the semiconducting sample gives rise to a current in the outer circuit, which is modified by an intensity grating that depends sinusoidally on space and time variables. In this section, we consider the induced constant current contribution calculated from $f_{1,0}$. Within a perturbation approach, we seek $f_{1,0} = \delta_{l}/l_{e}$ from Eq. (25) for $l=0$ and $p=0$ by using Eqs. (27) and (28) together with $N_{p,l}$ in Eq. (29). The quantity $N_{p,l}$ is factorized with respect to the frequency $\Omega = v_{gr}K_{e}$. For the constant current $\delta_{l0}$ induced by the static and moving interference patterns, we obtain the expression

$$\delta_{l0} = \frac{\alpha_{1}E_{0}}{2g_{0}^{2}(1 + \rho\sigma_{1})} \left\{ \frac{-g_{0}^{2}[1 + \kappa(\Lambda_{+} - \mu \Lambda_{-})]}{[1 + \alpha_{1}]^{2}} + \frac{g_{0}^{2}[\Omega_{M}(\Lambda_{-} - \Lambda_{+})]}{(\Omega_{M}^{2}[\Omega - \Lambda_{+}^{2}[\Omega - \Lambda_{-}^{2}])} \right\},$$

(30)

which consists of two contributions. The first one is due to the static grating and is proportional to the respective squared generation rate $g_{0}^{2}$. This contribution does not change its sign. When the electric field is switched off, this current component disappears. The second term on the right-hand side of Eq. (30), which describes the induced constant current created by the moving grating, is much more interesting. For a moving intensity modulation, the induced current $\delta_{l0}$ depends on the direction of motion with respect to the direction of the dc electric field and the type of conductivity of the sample. This current contribution may change its sign and continue to exist even when the external electric field has been turned off. The most salient feature of this term is the amplification effect associated with its pole structure. The poles result from SCWs that are resonantly excited, when their spatial and temporal periods coincide with the wavelength $\Lambda$ and the frequency $\Omega$, respectively, of the interference pattern. There are two quite different eigenmodes, the complex frequency of which are given by

$$\Omega_{1,2} = -\frac{1}{2}(\mu_{r}E_{0}K_{e} + i\Gamma) + \frac{1}{2}(\mu_{r}E_{0}K_{e} + i\Gamma) + \pm \frac{1}{4}(\mu_{r}E_{0}K_{e} + i\Gamma)^{2} + \frac{1}{\tau\tau_{M}(1 + \alpha)},$$

(31)

with

$$\Gamma = D_{s}K_{e}^{2} + \frac{1}{\tau} - \frac{1}{\tau_{M}},$$

(32)

and

$$\alpha_{1} = d_{s}(\Lambda_{+} - \mu \Lambda_{-}) + k\Lambda_{+}(1 - \mu^{2} + \Lambda_{+}^{2} - \Lambda_{-}^{2}) + \kappa \Lambda_{+} + ip\kappa[\mu - 2d_{s}(\Lambda_{+} - \mu \Lambda_{-})].$$

Eigenmodes of the $\Omega_{1}$ and $\Omega_{2}$ types are created by photoexcited trap electrons and have been discussed in the literature of SCWs in photorefractive crystals. However, the eigenmodes characterized by the dispersion relations in Eq. (31) refer to SCWs excited in a two-band system composed of electrons and holes. Two two-particle excitations are determined by the dynamics of electrons and holes. Under the conditions $|\text{Im}(\Omega_{1,2})| < |\text{Re}(\Omega_{1,2})|$, current resonances appear, whenever the moving grating resonantly excites one of the two SCWs. The character of these eigenmodes becomes more transparent under the condition $4(1 + \alpha)/(\tau\tau_{M}) < |\mu_{r}E_{0}K_{e} + i\Gamma|^{2}$, when we obtain

$$\Omega_{1} = -\mu_{r}E_{0}K_{e} - i\left(D_{s}K_{e}^{2} + \frac{1}{\tau} + \frac{1}{\tau_{M}}\right),$$

(33)

$$\Omega_{2} = \frac{1 + \alpha_{1}}{\tau\tau_{M}}\mu_{r}E_{0}K_{e} + i\left(D_{s}K_{e}^{2} + \frac{1}{\tau} + \frac{1}{\tau_{M}}\right).$$

(34)

The first mode $\Omega_{1}$ is associated with the dynamics of free carriers. Its linear dispersion relation is similar to the one of acoustic-phonon modes. This mode gives rise to a current resonance, when the grating moves synchronously with the SCW $[v_{gr}=(\mu_{r} - \mu_{p})E_{0}]$ under weak damping $\Gamma \ll v_{gr}K_{e}$. Resonant field amplitudes of this kind have been previously treated in a photoconductor. The second mode $\Omega_{2}$ remarkably differs from $\Omega_{1}$ and has the character of a trap-recharging mode, for which the phase and group velocities are oppositely directed. This mode gives rise to a current resonance, when the grating moves with the velocity $v_{gr} = (1 + \alpha_{1})/[\tau\tau_{M}(\mu_{r} - \mu_{p})E_{0}K_{e}^{2}]$ and the damping becomes weak ($\Gamma \ll v_{gr}K_{e}$). A numerical example for the field dependence of the induced constant current is shown in Fig. 2 for $T=4$ K as well as 77 K and parameters that are typical for experiments. The induced current $\delta_{l0}$ changes its sign at $E_{0}=0$. The excitation of SCWs with the frequencies $\Omega_{1}$ and $\Omega_{2}$ gives rise to two quite different current resonances. The first one with the character of an $\Omega_{1}$ mode appears at $E_{0} = 1.4$ K/cm and leads to a weak current maximum. With increasing temperature, this feature disappears. The second resonance of $\Omega_{2}$ type is more pronounced. It appears at $E_{0} = 0$, where the induced constant current changes its sign. At low temperatures, this resonance gives rise to an abrupt switching of the current direction.
In the absence of a dc electric field $E_0$, no resonant excitation of SCWs occurs, and the induced constant current is completely determined by relaxation processes. From Eq. (30), we easily obtain an expression for the induced current for $E_0 = 0$

$$\delta I_0 = \frac{2\pi^2}{e} \frac{(\mu_n + \mu_p)(D_n - D_p)K_s^2}{c_4} \frac{v_{gr}}{c_3^2 + c_2^2 + c_1}. \quad (35)$$

The same result has also been derived and used in the moving-photocarrier-grating technique$^{1,3}$ for materials characterization. In Eq. (35), we do not consider a load resistor, and the constants $c_2$, $c_3$, and $c_4$ are defined as in Refs. 3 and 4. Whenever $D_n \neq D_p$, the moving grating induces a constant current $\delta I_0$ that exhibits a maximum. By analyzing experimental data on the basis of Eq. (35), both the lifetime and the mobilities of photogenerated electrons and holes can be simultaneously determined.$^{2,7}$ In the lifetime regime, when $\tau_{M}/\tau \ll 1$ and the ambipolar diffusion length is much smaller than $K_{g}^{-1}$, the carrier recombination lifetime is directly obtained from the maximum velocity $v_{gr}^{\text{max}} = 1/(K_{g} \tau)$ corresponding to the maximum in the short-circuit current. We conclude that the well-established experimental method of materials characterization proposed in Refs. 2 and 3 rests on Eq. (35), which is obtained from the more general result in Eq. (30) in the limit of vanishing applied electric field $E_0 = 0$.

IV. INDUCED ALTERNATING CURRENT

In the widely used moving-photocarrier-grating technique for the determination of materials parameters,$^{2-7}$ the detected current is of dc type, which is also a disadvantage because of a reduction in the signal-to-noise ratio due to the low-frequency $1/f$ noise. A moving interference pattern alone as used in these studies cannot induce an alternating current. For this purpose, it is necessary to also provide a static grating, which has the same spatial periodicity. Due to the mutual influence of both intensity modulations, an alternating current is induced. In the same way as for the constant current treated in the previous section, the ac current is resonantly enhanced, when the spatial period and frequency of the intensity grating match the resonance condition for the SCW. We shall focus on the first harmonic of the induced alternating current. The related Fourier coefficients $f_{k1}$ are calculated from the basic Eqs. (24)–(26) together with Eqs. (27) and (28). Simple algebraic manipulations lead to the final result

$$\frac{\delta I_1(T)}{I_0} = 2 \text{Re} \, f_1 e^{i\omega t} = -\frac{g_{s} g_{m}}{2 g_{0}^{2}} \text{Re} \left[ \frac{e^{i\omega t}}{1 + \rho \tau_{p}(1 + i\Omega_{m})} \left( 1 - i\Lambda_{\omega} \right) \right] \times \left\{ 1 - \frac{1}{N_{1,1}} \left[ d_{\lambda}^{+} \left( \mu + i\kappa + \kappa \Lambda_{\omega} - \kappa \mu M_{1,1} \right) \right] \right\}
\quad + \left\{ 1 + i\Lambda_{\omega} \right\} \left\{ 1 - \frac{1}{N_{1,1}} \left[ d_{\lambda} \left( \mu + i\kappa + \kappa \Lambda_{\omega} \right) \right] \right\} \left\{ 1 + \frac{1}{N_{1,1}} \left[ d_{\lambda}^{+} \left( \mu + i\kappa + \kappa \Lambda_{\omega} \right) \right] \right\} \cdot \left\{ 1 - \kappa \mu M_{1,1} \right\}. \quad (36)$$

in which the following short-hand notation has been used

$$M_{\rho,\beta} = 1 + il\Omega \tau - ipd_{\rho}^{+} \mu + d_{\beta} \Lambda_{\omega}. \quad (37)$$

Resonances due to SCWs are described by the quantity $N_{-1,1}$ ($l = -1, p = 1$), which according to Eq. (29) is expressed by

$$N_{-1,1} = -\tau_{M} (\Omega_{1} - \Omega_{1}) (\Omega_{2} - \Omega_{2}). \quad (38)$$

where $\Omega_{1,2}$ are the complex frequencies of SCWs as defined in Eq. (31). When a weakly damped SCW is excited, a resonance is observed in the induced alternating current. An example of this resonance is shown in Fig. 3 for $T = 4$ K, where the same set of parameters has been used as for Fig. 2. The amplitude and phase of the alternating current sensitively depend on the strength of the applied electric field. There is an enhancement of the induced current due to the $\Omega_{2}$ mode accompanied by an abrupt phase shift at $E_0 \approx 0$. This enhanced phase switching of the alternating current is dominated by a current resonance at $E_0 \approx 1.4$ kV/cm, which results from an $\Omega_{1}$ mode. This amplification effect is much

\[\text{FIG. 2. Induced constant current } (1+\rho\tau_{p})\delta I_0 \text{ as a function of the electric field } E_0 \text{ for } \mu_n = 0.5 \text{ cm}^2/\text{Vs}, \mu_p = 0.2 \text{ cm}^2/\text{Vs}, \tau = 10^{-4} \text{ s}, g_{0} = 10^{19} \text{ cm}^{-3} \text{ (at } g_{0} = 0.5 g_{0}, \lambda = 633.8 \text{ nm, and } \alpha = 10^7). \text{ The grating moves with the velocity } v_{gr} = 700 \text{ cm/s}.\]

\[\text{FIG. 3. Induced alternating current } \delta I_1(t) \text{ as a function of the applied constant electric field } E_0 \text{ and the time variable } \Omega t \text{ for the lattice temperature } T = 4 \text{ K and } \mu_n = 0.5 \text{ cm}^2/\text{Vs}, \mu_p = 0.2 \text{ cm}^2/\text{Vs}, \tau = 10^{-4} \text{ s}, g_{0} = 10^{19} \text{ cm}^{-3} \text{ (at } g_{0} = 0.5 g_{0}, \lambda = 633.8 \text{ nm, and } \alpha = 10^7). \text{ The grating moves with the velocity } v_{gr} = 700 \text{ cm/s}. \text{ A load resistor was not considered (}\rho\tau_{p} = 0).\]
more pronounced in the induced ac than in the dc component shown in Fig. 2. Both SCW-induced current resonances survive also at higher temperatures, which is shown for $T = 77 \text{ K}$ in Fig. 4 from a different perspective as in Fig. 3. The induced ac current is displayed as a function of the electric-field strength $E_0$ and the time parameter $\Omega \tau$. Although the damping of SCWs increases with increasing temperature, the amplification effect of the ac current remains appreciable. We conclude that the current resonance induced by the SCW of $\Omega_1$ type is more robust for the ac component than for the dc one.

V. CONCLUSION

We have studied SCWs in semiconductors excited by the superposition of static and moving optical gratings. The treated SCWs are eigenmodes of oscillations in a semiconducting sample composed of excited electrons and holes that move in a constant electric field. A moving interference grating induces a constant current that continues to exist even when the applied electric field is switched off. If in addition to the moving pattern, a static optical grating is supplied, both dc and ac contributions are induced. These currents are resonantly enhanced, when the spatial and temporal periods of the interference pattern coincide with related quantities of the SCW and when the damping of SCWs can be neglected. There are two kinds of SCWs that remarkably differ from each other. One mode is due to the free-carrier motion. This SCW is resonantly excited when the photogenerated electron-hole pair moves synchronously with the optically generated moving interference pattern. The second oscillation mode has a group velocity that is oppositely directed to its phase velocity. This SCW has the character of a trap-recharging wave that plays an important role in the study of photorefractive crystals. The simultaneous detection of excited SCWs by dc and ac resonances provides complementary information that allows a detailed consideration of SCWs in semiconducting electron-hole systems. Our approach suggests an extension of the well-established moving-photocarrier-grating technique that has successfully been used for the determination of the lifetime and the mobilities of photogenerated electrons and holes in semiconductors. The extension encompasses on the one hand the application of an external constant electric field so that SCWs can be excited and on the other hand the simultaneous creation of static and moving intensity gratings so that both dc and ac current resonances are induced. Experiments in this direction would facilitate the study of SCWs in semiconductors.

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