

Quantum Hall ferromagnet at high filling factors: A magnetic-field-induced Stoner transition

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Spin splitting in the integer quantum Hall effect is investigated for a series of $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}$ heterojunctions and quantum wells. Magnetoresistance measurements are performed at mK temperature to characterize the electronic density of states and estimate the strength of many body interactions. A simple model with no free parameters correctly predicts the magnetic field required to observe spin splitting, confirming that the appearance of spin splitting, is a result of a competition between the disorder induced energy cost of flipping spins and the exchange energy gain associated with the polarized state. In this model, the single particle Zeeman energy plays no role, so that the appearance of this quantum Hall ferromagnet in the highest occupied Landau level can also be thought of as a magnetic field induced Stoner transition.

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I. INTRODUCTION

The integer quantum Hall effect (QHE),¹ observed in the magnetotransport properties of a two-dimensional electron gas (2-DEG) can be understood within the framework of a single electron picture. The magnetic field quantizes the orbital motion of electrons, so that the density of states consists of discrete, well-defined Landau levels, giving rise to the even-integer QHE. When the Zeeman energy is included, the spin degeneracy of each Landau level is lifted, giving rise to the odd-integer QHE. While the single particle picture works well for the even-integer QHE, this is not the case for the odd-integer QHE, where it misses most, if not all, of the essential physics. Following the pioneering theoretical work of Ando and Uemura,² the important contribution of exchange interactions to the spin gap is experimentally well established.³⁻⁵

Spectacular manifestations of many body effects have been extensively reported for low filling factors, notably at filling factor $\nu=1$, where the ground state is an itinerant quantum Hall ferromagnet,⁶ with an excitation spectrum which is either a spin wave⁷ or a spin texture excitation (Skyrmion).⁸ The appearance of a quantum Hall ferromagnet is analogous to the Stoner transition used to describe ferromagnetism in metals.⁹ In contrast, at high filling factors, in the limit of low magnetic field, the Shubnikov de Haas os-

cillations reveal that the spin up and down levels are degenerate, indicating that exchange interactions are not sufficiently strong to induce a ferromagnetic order within the highest occupied Landau level. In this work we investigate the transition between these two limiting behaviors. This transition is commonly referred to as the appearance of spin splitting. We show that this phenomenon can be understood by adapting the Stoner condition to take into account the modified two-dimensional (2D) density of states in a magnetic field.

In the most extensively investigated system, namely $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}$ heterostructures, the single particle Zeeman energy is rather weak (~ 0.3 K/T) owing to the small effective electronic g factor of GaAs ($g^* = -0.44$),¹⁰ whereas the experimental spin gaps evaluated, using, for example, thermal activation, are the order of ~ 6 K/T for the case of fully developed spin polarization at odd filling factor. This demonstrates the importance of the exchange enhancement of the spin gap, which is as a result often written as $g_{ex}^* \mu_B B$, where μ_B is the Bohr magneton and g_{ex}^* is an exchange-enhanced effective g factor first proposed in Ref. 11. The exchange-enhanced spin gap depends on the spin polarization of the 2-DEG,^{2,3} and therefore, this description is actually restricted to spin polarized odd filling factors (see, for example, the theory of Aleiner and Glazman).¹² When decreasing the magnetic field, experiments suggests that the

enhanced spin gap collapses, with odd-integer filling factor minima suddenly disappearing from the longitudinal resistivity $[\rho_{xx}(B)]$ at low fields.

A theoretical explanation for the collapse of spin splitting has been proposed by Fogler and Shklovskii,¹³ who predicted a second order phase transition in which exchange interactions are destroyed by disorder in the same way as temperature destroys ferromagnetism in the mean field theory. More precisely, when the spin gap attains the same order of magnitude as the disorder-induced Landau level broadening, the spin polarization at odd filling factor is reduced. This leads to the destruction of the polarization (exchange) part of the enhanced spin gap which can be qualitatively understood within the early description of Ando and Uemura.² Experimentally, the evolution of Fogler and Shklovskii's order parameter $\delta\nu$, which is the filling factor difference between two consecutive ρ_{xx} maxima related to spin up and down Landau levels, shows a universal behavior versus both temperature and electron density after judicious rescaling.^{14,15,5} Our experiments also confirm a collapse of the exchange-enhanced spin gap with decreasing magnetic field (Sec. IV B).

For a quantitative experimental description, parameters are needed to accurately describe the competition between exchange interactions and disorder. This is what is done in Ref. 5 in which disorder is extracted from an analysis of the low field Shubnikov de Haas oscillations, and exchange is estimated from transport measurements of the spin gap at higher fields, when spin splitting is fully resolved. From this an empirical critical filling factor is extracted for the onset of the collapse of spin splitting.

Here we propose to tackle the problem from an *intuitively* different point of view. Instead of starting from a spin polarized situation in which the spin gap is open as when discussing the collapse of spin splitting, we propose to tackle the phase transition from the other side, by considering what happens to an unpolarized (paramagnetic) 2-DEG, corresponding to the $\delta\nu=0$ phase in the Fogler and Shklovskii approach, when increasing magnetic field.

Spin splitting can then be seen as a transition from a paramagnetic state to a ferromagnetic state within a single Landau level at odd filling factors. For this transition to occur we have to compare the energy cost, due to the disorder-induced Landau level broadening, of populating higher energy levels by flipping spins [inversely proportional to the density of states at Fermi level, $D(E_F)$] and the exchange energy gain associated with the polarized state. At zero magnetic field this condition is nothing other than the well-known Stoner condition for ferromagnetism in metals.⁹ In a magnetic field, $D(E_F)$ increases as B/Γ , where Γ characterizes the Landau level broadening. Increasing magnetic field increases the density of states (Landau level degeneracy), thus lowering the energy cost for flipping spins sufficiently to induce a ferromagnetic state. For a quantitative comparison with experiment, estimations of the Landau level broadening Γ and the strength of the exchange energy are required.

The rest of this paper is organized as follows. In Sec. II a simple model for the appearance of spin splitting, based on the Stoner condition for ferromagnetism is presented. The Landau level broadening is evaluated in Sec. III from an

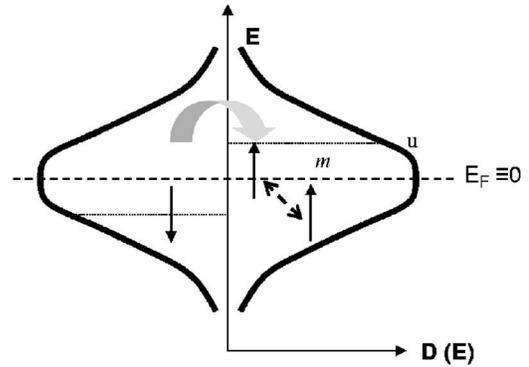


FIG. 1. Schematic representation of the process for spin splitting.

analysis of the low magnetic field Shubnikov de Haas oscillations. The exchange energy is derived in Sec. IV using the theoretical calculation of the electronic spin susceptibility of Attacalite *et al.*¹⁶ The calculated spin susceptibility and exchange parameter are shown to be in good agreement with experiment. Finally, in Sec. V, the Landau level broadening and exchange energy are used to predict the critical magnetic field for the appearance of spin splitting. A comparison with the experimental data shows good agreement for a number of samples covering a wide range of densities and mobilities.

II. MODEL FOR THE APPEARANCE OF SPIN SPLITTING

We propose here that exchange interactions drive a transition from a paramagnetic state, corresponding to nonspin-split Shubnikov de Haas oscillations, to a ferromagnetic state, corresponding to spin resolved Shubnikov de Haas. Strictly speaking, the transition is driven by the density of states, via the Landau level degeneracy eB/h , rather than the exchange energy, which is actually magnetic field independent, to a good approximation, at high filling factors (see Sec. IV B). The system is described considering the highest occupied Landau level at odd filling factor, so that the starting situation is when the system is unpolarized and the Fermi level lies in the center of the degenerate spin up and down sublevels of a given Landau level. This is an approximation, since there can be a small residual spin polarization due to the Zeeman splitting. However, in GaAs, at low magnetic fields, the Zeeman energy is small compared to both Landau level broadening and exchange energy, so it is a good approximation to consider the limit of zero Zeeman energy. Within this framework the appearance of a ferromagnetic state requires that the energy cost of populating higher energy levels by flipping spin, should be less than the gain in exchange energy, which therefore stabilizes the newly polarized state. This situation is depicted schematically in Fig. 1.

If we consider flipping spins to fill the spin up Landau level to an energy u above the initial Fermi energy, the energy cost can be expressed as

$$E_{flip} = 2 \int_0^u D(E) E dE. \quad (1)$$

To simplify, we have defined the zero of energy such that $E_F=0$. It is possible to obtain a simple expression if we as-

sume that the density of states, $D(E)$, in this expression can be approximated by its value at the Fermi level. Proceeding in this way, we implicitly suppose that the Landau level is rectangular shaped, which should be relevant only for small values of u . If the temperature is zero and $m=2\int_0^u D(E)dE \approx 2D(E_F)u$ is the spin polarization created by this process within Landau level N , then Eq. (1) can be written as

$$E_{flip} = \frac{1}{4D(E_F)}m^2. \quad (2)$$

The factor 4 in Eq. (2) simply arises from the definition of m . This expression confirms the intuitive idea that the larger the density of state is, the smaller the energy cost of flipping spins will be, simply because more states are available at lower energies. Calculations assuming a Gaussian form for the Landau level show this approximation differs from the exact result of Eq. (1) by less than 5%, provided $m < 0.425$.

At the same time this spin flip process leads to an increased population of the spin up Landau level, giving rise to an exchange energy gain which can be written

$$E_X = \frac{X_N}{4}m^2, \quad (3)$$

where X_N is the exchange energy between two spins in Landau level N . The quadratic dependence on m results from the many body nature of this interaction. Following this model in which we only have two competing energies, it will be energetically favorable for spins to flip when

$$\frac{1}{D(E_F)} = X_N. \quad (4)$$

When the Fermi level lies the center of a Landau level, $D(E_F)$ increases as B/Γ , due to the eB/h Landau level degeneracy and the disorder-induced broadening Γ . The underlying idea is that when increasing the magnetic field we reach a sufficient density of states at the Fermi level to reduce the spin flip energy cost, so that the spin system eventually becomes ‘‘ferromagnetic.’’

To check the validity of such a model, one must evaluate quantitatively how the density of states varies with magnetic field. For this a description of the shape and width of the Landau levels is required.

III. DENSITY OF STATES IN A MAGNETIC FIELD

A. Low field Shubnikov de Haas analysis

1. Formalism

In a magnetic field, the diagonal conductivity σ_{xx} of the 2-DEG can be written

$$\sigma_{xx} = \frac{n_s e^2 \tau_{tr}}{m^* (1 + \omega_c^2 \tau_{tr}^2)}, \quad (5)$$

where n_s is the electron density, τ_{tr} the transport scattering time, m^* the electron effective mass, and ω_c the cyclotron frequency.

In the limit of a zero temperature and if the magnetic field is large enough to satisfy $\omega_c \tau_{tr} \gg 1$, this simplifies to give

$$\sigma_{xx} \approx \frac{e^2 D(E_F)}{m^* \tau_{tr} \omega_c^2}. \quad (6)$$

For high mobility samples, the Hall resistivity is usually much larger than the longitudinal resistivity provided $\omega_c \tau_{tr} \gg 1$, so that the two-dimensional resistivity tensor gives $\rho_{xx} \approx \sigma_{xx} \rho_{xy}^2$. In the nonspin-split low magnetic field region, ρ_{xy} is approximatively linear in B , and from Eq. (6) the field dependance of ρ_{xx} can be written

$$\rho_{xx} \propto \frac{1}{\tau_{tr}} D(E_F). \quad (7)$$

At low magnetic field, the variation $\Delta\rho$ of the longitudinal resistivity around the zero field resistivity (ρ_0) can be calculated by expanding the quantized density of states as a Fourier series. The amplitude of the fundamental oscillatory term gives, in the zero temperature limit, the following expression for $\Delta\rho/\rho_0$:

$$\Delta\rho/\rho_0 = A e^{-\frac{\pi}{\omega_c \tau_q}}. \quad (8)$$

This expression, which is a good approximation provided that $\Delta\rho/\rho_0 < 1$, is obtained for the particular case of Lorentzian Landau level and is nothing more than the disorder damping term in the Lifshitz Kosevitch formula.¹⁷ Here τ_q is the quantum lifetime related to the width of the Lorentzian Landau level by $\tau_q = \hbar/2\Gamma_L$. A is a constant whose value depends on the power relation between the resistivity and the density of state (see below).

The same calculation for a Gaussian Landau level [defined here with a full width at half maximum of $2\sqrt{\ln(2)\Gamma_G}$] gives

$$\frac{\Delta\rho}{\rho_0} = A e^{-\pi^2 \Gamma_G^2 / \hbar^2 \omega_c^2}. \quad (9)$$

The expression is similar to that for a Lorentzian Landau level except that the B dependence in the exponential function is in $1/B^2$ if the Landau level width is field independent.

Plotting the experimental $\Delta\rho/\rho_0$ on a logarithmic scale versus $1/B$ or $1/B^2$ can give information concerning the shape of the Landau level. A linear behavior of $\ln(\Delta\rho/\rho_0)$ when plotted versus $1/B$ (so-called Dingle plot) indicates Lorentzian Landau levels,¹⁸ whereas a linear behavior when plotted versus $1/B^2$ suggests Gaussian Landau levels. In both cases the slope of these curves gives the (field-independent) width of these levels. This theoretical treatment has, however, to be confirmed by checking the value of the intercept at $1/B=0$, noted here as A . An intercept of $A=2$ is theoretically expected when the resistivity is directly proportional to $D(E_F)$. If, however, the ρ_{xx} dependence on $D(E_F)$ is quadratic [i.e., $1/\tau_{tr} \propto D(E_F)$ in Eq. (7)], the Fourier expansion exhibits an additional factor 2, which leads to an intercept of $A=4$. These issues are discussed in more detail in the semi-paper by Coleridge.¹⁹

2. Low temperature magnetotransport

We have performed mK temperature magnetotransport measurements on different heterojunctions (HJ) and quantum

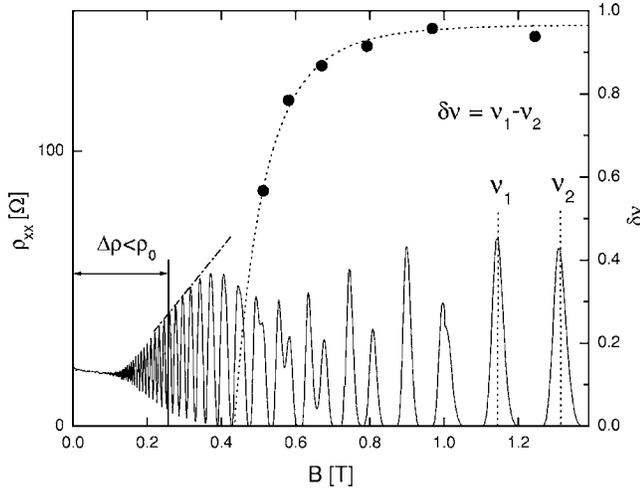


FIG. 2. Magnetoresistance trace for sample NU 1783a. $\delta\nu$ (full circles) as a function of magnetic field. Dotted line is the extrapolation of $\delta\nu$ used to determine B_{ss} as explained in Sec. V. Vertical solid line delimits the magnetic field region for which $\Delta\rho/\rho_0 < 1$. Dashed-dotted straight line emphasizes the linear behavior of $\rho_{xx}^{peak}(B)$ at higher magnetic fields.

wells (QW), using a standard low frequency lock-in technique. Representative longitudinal resistivity (ρ_{xx}) versus magnetic field data obtained for one of the samples is shown in Fig. 2. The variation of the longitudinal resistivity around the zero field value ($\Delta\rho/\rho_0$) has been directly extracted from the low field Shubnikov de Haas data at 50 mK. ($\Delta\rho/\rho_0$) is plotted on a logarithmic scale versus $(1/B^2)$ in Fig. 3(a), and, for comparison, versus $(1/B)$ in Fig. 3(b), for all the samples investigated.

At $T=50$ mK, the smearing imposed by the Fermi-Dirac statistics is small enough to ensure that $\Delta\rho$ can be analyzed using a zero temperature formalism. In the samples studied, the Dingle plots are found to decrease as $1/B^2$ rather than as $1/B$ (see Fig. 3). While the linear fits are reasonable in both cases, they are slightly better in the $1/B^2$ plots [Fig. 3(a)]. The data in the $1/B$ plots has a noticeable curvature [Fig. 3(b)]. In addition, the intercepts in the $1/B^2$ plots ($1 < A < 2.7$) are close to one of the theoretically expected values, $A=2$. This is not the case for the $1/B$ plots, with intercepts $7 < A < 21$, far from the theoretically expected value of either $A=2$ or $A=4$.

This suggests that, for our samples, the density of states is better described using a Gaussian form with field-independent width for the Landau levels. Within this model, the intercept $A \approx 2$, implies that ρ_{xx} is directly proportional to $D(E_F)$. We have also checked that an identical result is obtained when filtering the raw data with a band pass centered at the fundamental Shubnikov de Haas frequency. The $1/B^2$ decrease (a Gaussian form for the Landau levels) therefore provides a better description of our data. From Eq. (7) this result implies, as proposed in Ref. 20 that the transport scattering time does not depend on $D(E_F)$.

A complementary study to the low field Shubnikov de Haas analysis can be performed by analyzing the values of the resistivity peaks, $\rho_{xx}^{peak}(B)$, in the regime $\Delta\rho/\rho_0 > 1$.^{20,21}

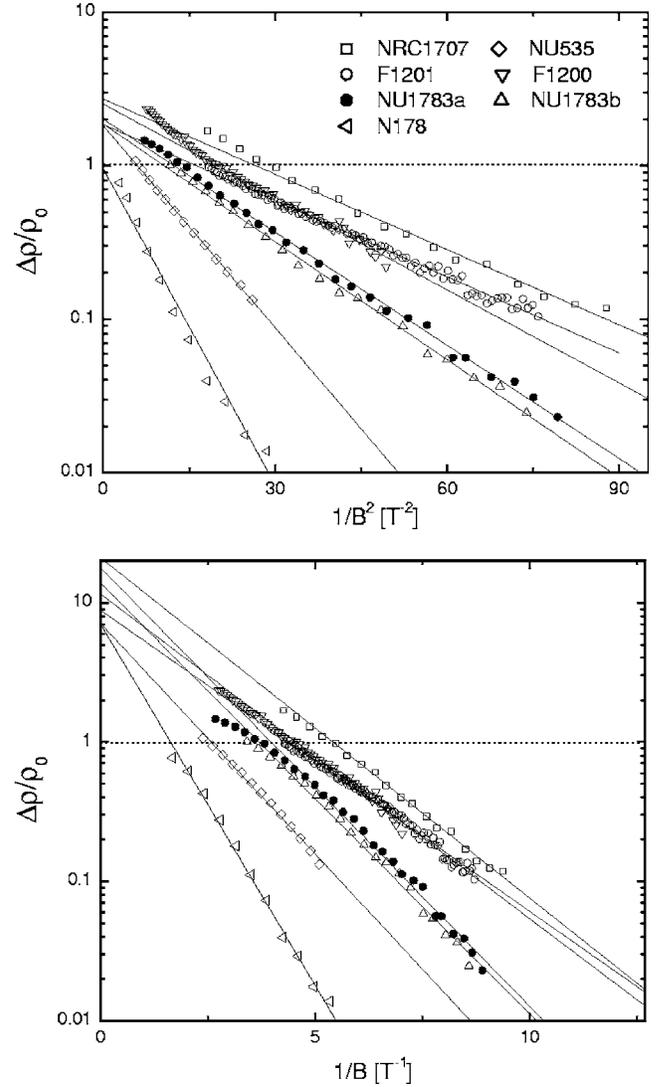


FIG. 3. Dingle plots. (a) $\Delta\rho/\rho_0$ versus $1/B^2$ and (b) $\Delta\rho/\rho_0$ versus $1/B$. Note log scale for vertical axes. Straight lines are linear fits to the data points in the region $\Delta\rho/\rho_0 < 1$ delimited by the horizontal dotted line.

In this regime, we observe a linear increase of $\rho_{xx}^{peak}(B)$ (visible in the ρ_{xx} trace for sample NU1783a in Fig. 2). When $\Delta\rho/\rho_0 > 1$, Eqs. (8) and (9) are no longer valid because the deviation of the density of states from the zero field value becomes too large to be described by the fundamental oscillatory term in the Fourier series. However, Eq. (7) is still valid and thus the field dependence of the resistivity peaks can provide information on $(1/\tau_r)D(E_F)$. The less stringent condition to account for the observed linear dependence of $\rho_{xx}^{peak}(B)$, is as proposed in Ref. 20, that the inverse of the transport relaxation time and the Landau level broadening have the same magnetic field dependence. For the sample studied in Ref. 20, this dependence was in square root of B , while here the low field Shubnikov de Haas (Dingle) analysis suggests both parameters are field independent. The latter case is fully compatible with the observed linear dependence of $\rho_{xx}^{peak}(B)$.

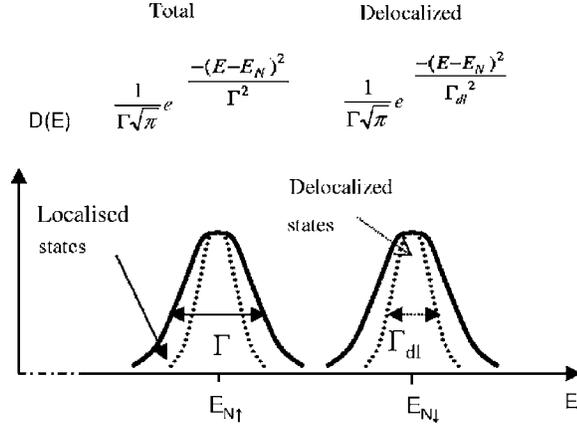


FIG. 4. Schematic representation of the density of states in a magnetic field used in a simple model to estimate Γ in Sec. III.

The theory of Raikh and Shahbazyan²² on the Landau level shape in the presence of long range scattering suggests Gaussian Landau levels with field-independent width in the regime where $a_{corr} \gg R_{cycl}$ (where a_{corr} is the correlation length of the disorder potential, and R_{cycl} the cyclotron radius) and with a square root of B dependence when $R_{cycl} \gg a_{corr}$. Our results are consistent with a field-independent broadening of the Gaussian Landau levels suggesting that $a_{corr} \gg R_{cycl}$ in our samples. While this is an interesting point discussed in Refs. 19 and 22, it is not the direct focus of the present work.

B. Estimation of the Landau level width

To evaluate the $D(E_F)$ appearing in Eq. (4), one needs an estimation of the total Landau level width Γ , which includes all states, both delocalized and localized. Estimating Γ from the low field Shubnikov de Haas oscillations is *a priori* not possible because in this region delocalized states from different Landau levels overlap, preventing the resistance from going to zero. However, at slightly higher fields, in the region just before spin splitting appears, the proximity to the first zero resistance states at even filling factors can be used to estimate Γ . The width of the ρ_{xx} minima (zero resistance states) depends on the ratio Γ_{dl}/Γ , where Γ_{dl} characterizes

the width of the extended (delocalized) states.

Starting from the formalism presented in Sec. III A, we construct a simple model in the zero temperature limit, assuming that the density of states can be modeled as a set of Gaussian functions of width Γ , with sub-Gaussians delimiting delocalized states near the center of the Landau level, of width Γ_{dl} . This is schematically depicted in Fig. 4. $\Gamma_{dl} \equiv \Gamma_G$ is determined from the slope of the $\ln(\Delta\rho/\rho_0)$ versus $1/B^2$ plots as described in Sec. III A. The values of Γ_{dl} for all the samples investigated are summarized in Table I. ρ_{xx} can then be calculated using $\rho_{xx} \propto D(E_F)$. Knowing Γ_{dl} , it is possible to determine Γ for each sample, by fitting to the $\rho_{xx}(B)$ data in the magnetic field region just before spin splitting appears. Proceeding in this way we have estimated Γ for the different samples studied (see Table I). Experimentally a Gaussian form for the density of delocalized states gives by far the best fit to our $\rho_{xx}(B)$ data. We have also tried to fit assuming the region of delocalized states is delimited with a sharp cutoff (mobility edge), as often proposed in the literature. While this model is unable to correctly reproduce the shape of $\rho_{xx}(B)$ we can nevertheless roughly estimate Γ . The results are comparable to those obtained with our Gaussian model, suggesting that the Landau level width Γ extracted here is to some extent independent of the exact form used for the density of delocalized states.

IV. ESTIMATION OF THE EXCHANGE INTERACTION

Analyzing the spin-split Shubnikov de Haas oscillations we can extract parameters which characterize exchange, such as, for example, the enhanced-effective g factors.⁵ However, in this case the spin gap already exists, and, the system is therefore already polarized. To describe an unpolarized 2-DEG in a low field situation, it is better to estimate exchange via the spin susceptibility of the electronic system, defined as the response to an external magnetic field in terms of spin magnetization. This is a good probe of exchange interactions between electrons since exchange favors spin alignment and thus enhances the spin susceptibility. We will see in Sec. IV B that these two different approaches to estimate the exchange (enhanced-effective g factors and spin susceptibility) are fully consistent.

TABLE I. Samples parameters: structure, electron density (n_s), mobility at 50 mK (μ), width of delocalized states (Γ_{dl}), total Landau level width (Γ), magnetic field for the appearance of spin splitting (B_{ss}) discussed in Sec. V and exchange-enhanced g factor (g_{ex}^*) discussed in Sec. IV B. For N 178 a reliable estimation of Γ was not possible.

Sample	Structure	n_s (10^{11} cm^{-2})	μ ($10^6 \text{ cm}^2/\text{Vs}$)	Γ_{dl} (K)	Γ (K)	B_{ss} (T)	g_{ex}^*
F1201	QW	8.83	1	1.23 ± 0.05	2.3 ± 0.1	0.93 ± 0.07	5.3 ± 1.5
F1200	QW	7.55	1	1.35 ± 0.05	1.9 ± 0.1	0.72 ± 0.1	5.7 ± 0.4
NRC1707	HJ	1.64	5.8	1.1 ± 0.05	1.5 ± 0.1	0.26 ± 0.05	13.1 ± 1
NU 1783a	HJ	2.1	1.5	1.48 ± 0.05	2.2 ± 0.1	0.43 ± 0.06	13 ± 1
NU 535	QW	2.13	0.11	2 ± 0.05	3.1 ± 0.2	0.74 ± 0.12	9.5 ± 1
NU 1783b	HJ	1.77	1.8	1.53 ± 0.05	2.3 ± 0.2	0.32 ± 0.11	13.7 ± 0.7
N 178	HJ	1.32	0.32	2.5 ± 0.05	–	0.54 ± 0.2	16 ± 1.5

A. Spin susceptibility

1. Theoretical approach

At zero magnetic field, the nature of the 2-DEG ground state is strongly dependent on the electron density, and thus often described as a function of the so-called density parameter r_s , corresponding to the ratio of the Coulomb energy to the kinetic energy. In 2D, r_s scales as $1/\sqrt{n_s}$. At high r_s , in the low density regime, the ground state remains a topic of controversy, with a possible manifestation of a metal to insulator transition (MIT) observed around $r_s \sim 10$.²³ At low r_s however, and especially for the density range investigated here ($r_s < 2$), the density is sufficiently high to ensure that the zero field ground state is a paramagnetic liquid. In this case, the paramagnetic relative spin susceptibility χ/χ_0 of the 2-DEG can be derived from calculations of the $B=0$ ground state energy as a function of density and polarization, for example, using quantum Monte Carlo simulations (see, e.g., Tanatar and Ceperley²⁴). Here, χ/χ_0 corresponds to the ratio of the “real” spin susceptibility of interacting electrons, χ , to the noninteracting Pauli value χ_0 . Similar calculations have been performed more recently by Attacalite *et al.*¹⁶ Different predictions arise from the different ways of estimating the total energy of the ground state, and in particular the correlation term. At low density, χ/χ_0 turns out to be a very delicate quantity to estimate, and the various predictions are significantly different.

Fortunately, for our rather high density systems ($r_s < 2$), the behavior of the predicted susceptibilities is more robust. Indeed, predictions for small values of r_s do not show large discrepancies because in the limit $r_s \rightarrow 0$ they all reach the “basic” Hartree-Fock approximation for spin susceptibility, determined using the well-known Hartree-Fock 2D ground state energy ($E_g = 1/r_s^2 - 1.2004/r_s$, see, e.g., Ref. 25). This Hartree-Fock unscreened susceptibility increases with increasing r_s , reflecting that the role of exchange interactions becomes more important at low density. This trend is common to a large majority of predictions over a very large r_s domain. However, the Hartree-Fock susceptibility diverges for $r_s \sim 2.2$ because it does not include the effects of the correlation energy and/or screening, which prevents χ/χ_0 from diverging. Taking into account these corrections, as done for instance in Ref. 16, the r_s increase is still present but much smoother. These different theoretical predictions are plotted for comparison in Fig. 5.

2. Experimental approach

There are several experimental techniques to measure the electron spin susceptibility of a 2-DEG (for a review see Ref. 26). For the particular case of a 2-DEG in GaAs, significant results have recently been obtained with the tilted field method by Zhu *et al.*²⁷ The principle of this method is to induce Landau level coincidences by tilting the sample in a magnetic field. When tilting the sample, the magnetic field perpendicular to the 2-DEG is reduced (see inset of Fig. 6). We recall that the Zeeman gap depends on the total magnetic field, whereas the cyclotron gap, which is an orbital effect, depends only on the field component perpendicular to the 2-DEG.

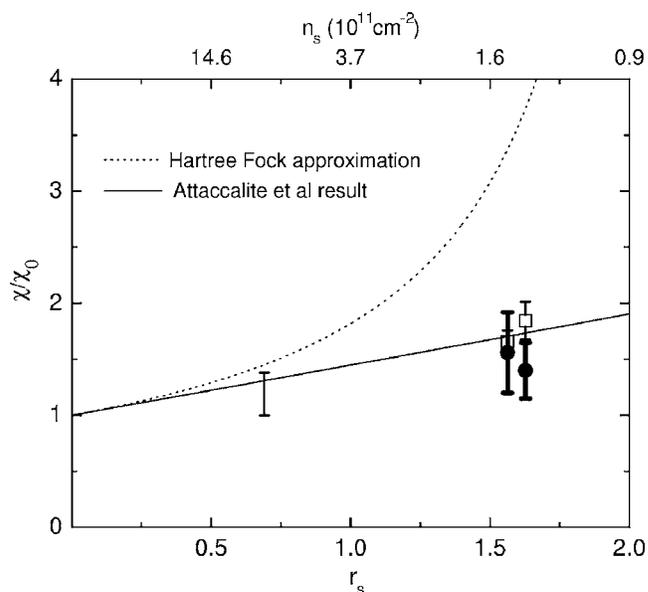


FIG. 5. Theoretical spin susceptibility as a function of the density parameter r_s . Hartree-Fock approximation (dotted curve), Attacalite *et al.* prediction (solid curve). Open squares are experimental values of χ/χ_0 at $i=1$ determined by tilted-field experiments on different samples. For the high density sample ($r_s \approx 0.7$) only an upper limit could be extracted. Full circles are extrapolated ($B \rightarrow 0$) values of the paramagnetic χ/χ_0 .

Therefore, by tilting the sample, for a given filling factor, we can increase the relative weight of the Zeeman gap until the point where the spin gap is equal to the cyclotron gap. This is what we refer to here as the coincidence condition (note in the literature coincidence condition often refers to the case where the spin gap is half the cyclotron energy). Knowing the angle θ (defined in the inset of Fig. 6) which corresponds to this situation, the product $m^* g_{\text{tilt}}$, satisfying $m^* g_{\text{tilt}} = i 2 m_e \cos(\theta)$, where $i=1$ can be extracted. If normalized using the band structure parameters ($m^* = 0.068 m_e$, m_e being the electron rest mass, and $|g^*| = 0.44$), this $m^* g_{\text{tilt}}$ is directly equal to the relative spin susceptibility χ/χ_0 at $i=1$.

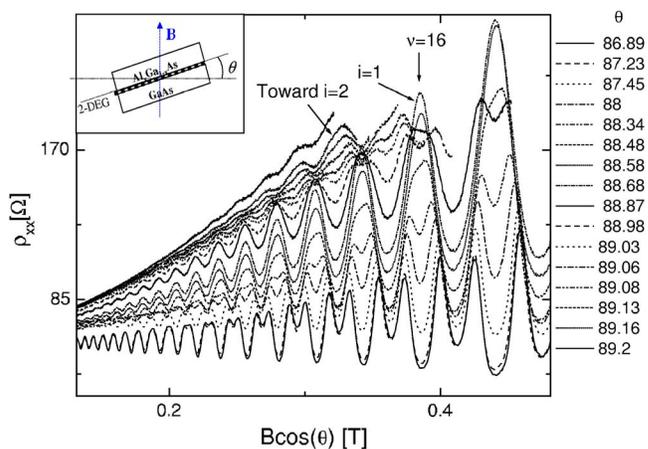


FIG. 6. (Color online) Magnetoresistance traces for sample NU 1783b at different tilting angles. Angles are determined from the slope of the low field Hall resistance.

The parameter g_{tilt}^* describes the total spin gap assuming it can be written $g_{\text{tilt}}^* \mu_B B$ where B is the total magnetic field consistent with the definition of the spin susceptibility. The enhanced g factor can also be defined using the perpendicular component of the magnetic field, related to the two-dimensional (orbital) nature of exchange.⁵

Proceeding with a tunable density gated sample, Zhu *et al.*²⁷ give an explicit density-polarization dependence of the relative susceptibility for a range of density going from 2×10^9 to 4×10^{10} cm⁻². The polarization dependence is obtained from the observed i -dependence of χ/χ_0 , because different values of i correspond to a different polarization of the system. The limit of zero polarization can provide an estimation of the paramagnetic relative susceptibility of the 2-DEG. Their results show the experimental χ/χ_0 is much smaller than the Hartree-Fock prediction and also that the r_s trend is in good agreement with the Attacalite *et al.* calculation, although the experimental values are slightly smaller than predicted. Other experimental data on different systems, mainly 2-DEG in Si, can complete this comparison if they are rescaled in terms of r_s (see, e.g., Ref. 26), and show that this trend is also respected in the lower r_s region.

To check if the Attacalite *et al.* prediction is experimentally valid for GaAs in the region of interest here ($r_s < 2$), we have performed tilted-field measurements on our samples which cover a higher density range than the one in Ref. 27. Going towards high density should reduce spin susceptibility and thus requires even larger tilt angles, and thus higher magnetic fields. Transport measurements were performed using standard low frequency lock-in technique under magnetic fields up to 23 T, in a dilution fridge (base $T \approx 50$ mK) equipped with an *in-situ* rotating sample holder.

In Fig. 6 we plot $\rho_{xx}(B)$, measured at $T=50$ mK, and at various tilt angles, for sample NU1783b. Note different traces are “naturally” shifted upwards, due to the effect of the increasing in plane magnetic field which leads to a positive magnetoresistance.²⁸ The angles were accurately determined from the slope of the low magnetic field Hall resistance. As expected, we find that coincidence is reached for even larger tilt angles than in the Zhu *et al.* experiment.²⁷ This is consistent with previous measurements in this higher density range.⁵ For example, in sample NU1783b coincidence is reached at approximately 88.7° (see Fig. 6). The fact that we can use tilted field to tune the Zeeman energy is not inconsistent with our simple model in which the single particle Zeeman energy is assumed to be negligible in the $\theta=0^\circ$ configuration. The large tilt angle required implies that an in-plane magnetic field approximately two order of magnitude larger than the perpendicular magnetic field component is necessary to achieve Landau level coincidence.

If we consider the situation at even filling factor, for example, at $\nu=16$ indicated by the arrow in Fig. 6, we see that when increasing θ the gap at even filling factor initially closes, until the spin up and down sublevels of adjacent Landau levels coincide, here at about 88.7 degrees. Tilting further, we are able to make the levels cross, confirming we do indeed obtain a coincidence of Landau levels. We can see that the gap reopens at even filling factors, and then closes at odd filling factors, probably reaching a second coincidence in which the spin gap is twice the cyclotron gap. The coin-

idence condition depends only weakly on the perpendicular magnetic field. In other words, a single tilt angle θ provides coincidence nearly simultaneously at all observed even filling factors. This means that the polarization dependence of χ/χ_0 is weak, more precisely sufficiently weak, that its effects are suppressed by Landau level broadening.

Determining the coincidence condition from the ρ_{xx} trace, the corresponding products $m^* g_{\text{tilt}}^*$ have been extracted for two samples measured (1783b and N178). For the high density samples, coincidence occurs at higher tilt angles ($\theta > 88.81^\circ$), and, due to sample mobility and magnetic field limitations, coincidence cannot be reached. Nevertheless, we can extract an upper limit for the product $m^* g_{\text{tilt}}^*$ of sample 1200, showing that the susceptibility is indeed slightly lower for this higher density sample. The values of $m^* g_{\text{tilt}}^*$, normalized by the product of the band structure values $m^* = 0.068 m_e$ and $|g^*| = 0.44$, are plotted versus r_s in Fig. 5 (open squares).

The paramagnetic spin susceptibility χ/χ_0 can then be estimated from the $i=0.5, 1$ (and 1.5 when it is observed) conditions at a given perpendicular magnetic field, assuming the i dependence of spin susceptibility is linear as observed in Ref. 27. Such a linear extrapolation induces large error bars, because in the best situation we only have three values for $\chi/\chi_0(i)$, themselves subjected to error. As the tilt angle required for different coincidence conditions is roughly the same for all filling factors, extrapolation at different perpendicular magnetic field gives similar results. The differences are within the error bars induced by the linear extrapolation. While, the value of χ/χ_0 at $i=1$ is subject to less uncertainty than the extrapolated paramagnetic χ/χ_0 , rigorously it is the latter which is required for a comparison with theory.

The paramagnetic χ/χ_0 are plotted versus r_s for samples 1783b and N178 in Fig. 5 (closed circles). They are consistent with the Attacalite *et al.* prediction, smaller at most by 20%. The experimental values in Ref. 27 are also smaller than the theoretical prediction. However, the calculation of Attacalite *et al.* is performed for an ideal 2-DEG, whereas experimentally the finite thickness of a real 2-DEG is expected to reduce spin susceptibility. Very recent finite thickness corrections to the Attacalite *et al.* calculation have been performed by DePalo *et al.*,²⁹ showing these effects indeed lead to a reduction of the theoretical values, reconciling them with experiments (see also Ref. 30). Furthermore, spin susceptibility measurements on system similar to the one used in Ref. 27 have been recently extended to the high density regime down to $r_s=0.8$,³¹ showing good quantitative agreements with the finite thickness correction in this density regime.²⁹

What is important for this work is that the Attacalite *et al.* calculations provide a relevant estimation of spin susceptibility in our samples, and therefore we can use this theory to derive our exchange parameter.

B. Exchange parameter versus density

We now have to draw a formal link between the relative spin susceptibility and the exchange parameter X_N defined in Sec. II. As χ/χ_0 is calculated for the case of zero magnetic

field, our condition for the appearance of spin splitting should be applied in the limit $B=0$, which gives $X_\infty D(E_F)_{B=0}=1$.

As this condition corresponds to a zero-field “ferromagnetic” state, we can define

$$\frac{\chi}{\chi_0} = \frac{1}{1 - X_\infty D(E_F)_{B=0}}. \quad (10)$$

Here $D(E_F)_{B=0}$ is the zero field density of states of the 2-DEG, for one spin orientation, i.e., $m^*/2\pi\hbar^2$. Here m^* is taken to be equal to the accepted GaAs band edge value of $0.068m_e$. The zero field exchange parameter X_∞ is then obtained from the Attacalite *et al.* spin susceptibility $(\chi/\chi_0)_{Att}$ as a function of density,

$$X_\infty(n_s) = \frac{\left(\frac{\chi}{\chi_0}\right)_{Att}(n_s) - 1}{\left(\frac{\chi}{\chi_0}\right)_{Att}(n_s) D(E_F)_{B=0}}. \quad (11)$$

In order to compare this prediction with experiment, we have estimated the exchange-enhanced effective g factors g_{ex}^* . Starting from the density of states, $D(E)$, as described in Sec. III, we use a simple zero temperature model, in which $D(E)$ is described by a set of Gaussian Landau levels with magnetic field-independent width.³² The longitudinal resistivity ρ_{xx} is calculated using $\rho_{xx} \propto D(E_F)$. We have extracted the value of g_{ex}^* by fitting odd filling factor minima in ρ_{xx} assuming that the Landau level broadening involved here is the same as the one extracted from the low field Shubnikov de Haas oscillations. This assumption, also used in Ref. 5, supposes that the field independence of the Landau level width, observed at low field, still holds in the magnetic field region where spin splitting first appears. This field-independent Landau level width is suggested by the $1/B^2$ dependence of $\ln(\Delta\rho/\rho_0)$ in the Dingle plots (Fig. 3).

As expected, a single electron picture, in which the spin gap is taken to be equal to the Zeeman gap, fails to reproduce the observed spin splitting. It is, however, possible to reproduce odd filling factor minima using a magnetic field (filling factor) dependent phenomenological spin gap, $g_{ex}^* \mu_B B$, where g_{ex}^* takes into account the exchange enhancement of the spin gap. Representative results can be seen in Fig. 7, where we plot $\rho_{xx}(B)$, measured at $T=50$ mK, for one of the samples. The broken lines are the simulated $\rho_{xx}(B)$ calculated for each odd filling factor using the g_{ex}^* indicated in the legend. The exchange-enhanced effective g factor collapses when approaching the nonsplit ρ_{xx} region, consistent with the Fogler and Shklovskii phase transition,^{5,13} reflecting the disorder-induced destruction of spin polarization at odd filling factor.

When the spin gap is fully open, the spin polarization is a maximum and the value of g_{ex}^* (reported in Table I) can be compared with theory. Neglecting the single particle Zeeman energy, we can equate the exchange gap, $X(eB/h)$, to the total spin gap $g_{ex}^* \mu_B B$. This corresponds to the fully polarized case in which a spin interacts with the eB/h electrons in the lower spin Landau level. The values of $X = g_{ex}^* \mu_B (h/e)$ for the

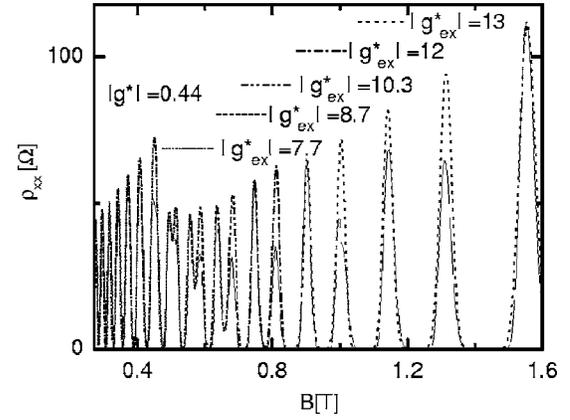


FIG. 7. Measured ρ_{xx} versus magnetic field for sample NU1783a at $T=50$ mK (solid line). Calculated ρ_{xx} taking into account exchange interactions with a phenomenological spin gap $g_{ex}^* \mu_B B$ (broken lines).

different samples studied are plotted in Fig. 8 (closed circles) together with the predicted value X_∞ obtained by Eq. (11) (solid line). As previously observed in $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}$ (see, e.g., Ref. 5), g_{ex}^* is larger in low density samples. The estimation derived by Eq. (11) from Attacalite *et al.* spin susceptibility is in good quantitative agreement with the one obtained from our g_{ex}^* , even though the exact functional dependence upon r_s (sublinear) is not well reproduced by the data.

For comparison, we also show in Fig. 8 other theoretical estimations of the exchange parameter. Using the Ando and Uemura² exchange-enhancement of the spin gap, we can again write $X(eB/h) = 1/\nu(e^2 k_F/4\pi\epsilon)$. Here the magnetic length has been replaced by $1/k_F$ which is the relevant length scale at high filling factors.^{5,12} The filling factor $\nu = hn_s/eB$

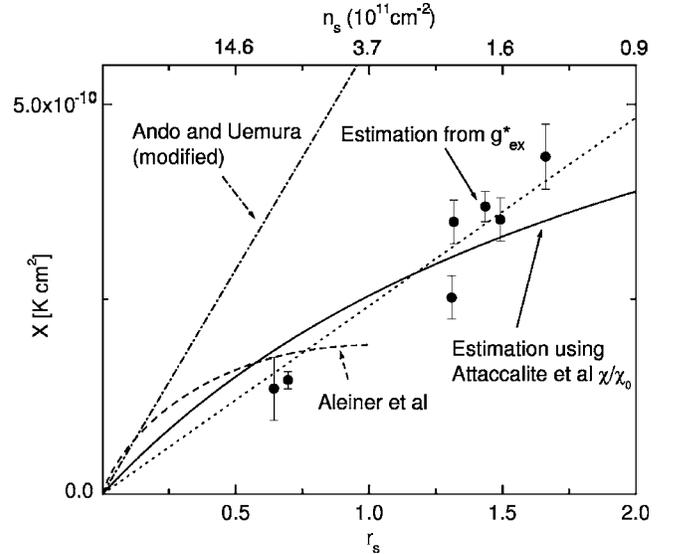


FIG. 8. Exchange parameters as a function of the density parameter r_s , and electron density n_s (note nonlinear scale for the top axis). Estimation from Ando and Uemura² (dashed-dotted line), from Aleiner *et al.*¹² (dashed line) and from the experimentally determined g_{ex}^* (full circles). The dotted line is a linear fit to $g_{ex}^*(r_s)$.

and $k_F = \sqrt{2\pi n_s}$ so that in Fig. 8 we plot $X = e^2 \sqrt{2\pi/n_s} / 4\pi e$ (dotted-dashed line). The dependence on $1/k_F$ rather than l_B leads to a $1/\sqrt{n_s} \propto r_s$ density dependence. It is striking how this simple argument describes qualitatively the density dependence of our enhanced g factor $g_{ex}^* \propto r_s$ [as visible on the linear fit to $g_{ex}^*(r_s)$ in dotted line]. We have also estimated the exchange parameter from the α parameter describing the exchange gap in Ref. 12, which is valid for $r_s < 1$ (dashed line). The rigorous calculation,¹² including screening, introduces a logarithmic correction which reduces the exchange parameter at higher r_s , improving agreement with experiment.

Finally we note that the influence of disorder on the measured g_{ex}^* seems to be weak since different samples with different Landau level broadening keep within the density trend. This is consistent with the fact that the effect of disorder on the spin susceptibility extracted by the tilted-field method is known to be weak.²⁶ To summarize, the good agreement observed between experiment and theory means that the exchange parameter for our samples can be reliably estimated using Attacalite *et al.* spin susceptibility. To a good approximation, the X_N appearing in Eq. (4) can be identified with its zero field value, X_∞ , given by Eq. (11). The absence of a magnetic field dependence can be understood, since at high filling factors (low B) the average electronic separation is $1/k_F$, rather than the magnetic length.

V. CRITICAL MAGNETIC FIELD FOR SPIN SPLITTING

A. Predicted field for spin splitting

From Eq. (4) in Sec. II, we can directly obtain a relation which explicitly gives the magnetic field B_{ss} at which the spin degeneracy of a Landau level should be lifted, as a function of the Landau level broadening Γ , and the electron density n_s ,

$$B_{ss} = \frac{\hbar\Gamma\sqrt{\pi}}{e} \frac{1}{X_\infty(n_s)}, \quad (12)$$

where X_∞ is given by Eq. (11). We see here that B_{ss} is predicted to increase with Γ , so that spin splitting appears later in more disordered samples, which is also experimentally well established. The same Γ dependence is found for the condition for the collapse of spin splitting when comparing the spin gap to the Landau level broadening.^{5,13} The electron density dependence of B_{ss} arises via the dependence of X_∞ on n_s . With X_∞ defined as in Eq. (11) this gives for B_{ss} a dependence close to $\sqrt{n_s}$, pushing spin splitting to higher magnetic field for higher density samples.

B. Comparison with experiment

Experimentally, the spin-splitting phenomenon is probed by the presence of a minimum appearing at odd filling factor in the longitudinal resistance. A method of extracting B_{ss} from the experimental data is needed for a quantitative comparison. In Ref. 13, the critical filling factor (magnetic field) for collapse of spin splitting is defined as the filling factor corresponding to $\delta\nu=0.5$. Here $\delta\nu$, is the filling factor difference between two consecutive ρ_{xx} maxima related to spin

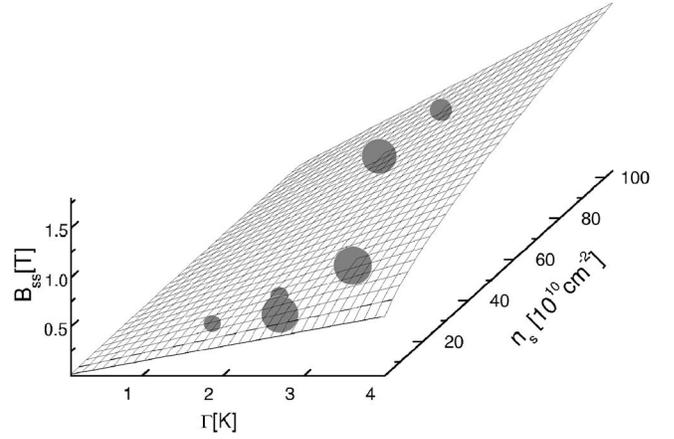


FIG. 9. Magnetic field for spin splitting (B_{ss}) plotted versus Landau level broadening (Γ) and density (n_s). The wire frame is the prediction of Eq. (12), and the spheres are the experimentally determined B_{ss} . The experimental error is indicated by the size of the spheres.

up and down Landau levels. This method, has been successfully applied to analyze the ‘‘collapse’’ of spin splitting, in which the spin gap is compared to the disorder in order to determine when the spin splitting should collapse.⁵

In a similar manner, we have analyzed our $\rho_{xx}(B)$ data to extract $\delta\nu$, as shown in Fig. 2 for sample NU1783a. In our approach, in which we start from the ‘‘paramagnetic’’ state, the condition $\delta\nu=0$ is more appropriate. As the condition $\delta\nu=0$ cannot be accessed experimentally, we have to extrapolate from the experimental $\delta\nu(B)$, for which we typically have data in the regime $0.5 \leq \delta\nu \leq 1$ (see Fig. 2). The value of B_{ss} determined in this manner is only slightly smaller than if we had used the $\delta\nu=0.5$ criterium, and the error implied by such a treatment is estimated for each sample from the width of the transition.

Rigourously, as our model is developed for the zero temperature case, what is required is the extrapolated value of B_{ss} at zero temperature. However, we have performed this analysis at different temperatures ($0.05 \leq T \leq 1.2$ K) and the extracted evolution of B_{ss} versus temperature shows that the difference between the extrapolated $T=0$ K value, and the 50 mK value, never exceeds 5%, which is well within the error bar for B_{ss} .

In Fig. 9, B_{ss} is plotted both versus Landau level broadening (Γ) and electron density (n_s), for the six different samples (HJ or QW), spanning a density and mobility range respectively of $\sim(1.5-9) \times 10^{11} \text{ cm}^{-2}$ and $\sim(0.1-6) \times 10^6 \text{ cm}^2/\text{V s}$. The Γ values, which are also given in Table I, have been estimated as explained in Sec. III. To compare experiment with our model, the predicted evolution of B_{ss} calculated using Eq. (12), is plotted as a wire mesh (three-dimensional plot) in Fig. 9. The exchange energy X_∞ has been estimated using the calculated spin susceptibility of Attacalite *et al.*¹⁶ as explained in Sec. IV. Figure 9 shows a good quantitative agreement between the prediction and the experimental B_{ss} , especially considering that there are no adjustable parameters in the model. If we focus on the B_{ss} variation at constant disorder, the density dependence is cor-

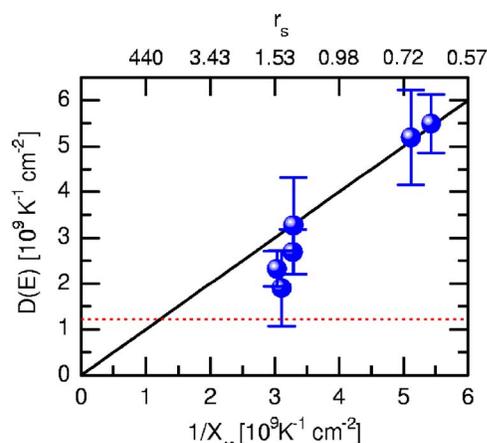


FIG. 10. (Color online) Critical density of states at the Fermi level [$D_{ss}(E_F)$] for a paramagnetic-ferromagnetic transition plotted as a function of the inverse exchange energy $1/X_\infty$. The $D_{ss}(E_F)$ predicted as explained in the text using Eq. (12) (solid line) and the zero field 2D density of states (broken line) are also plotted for comparison.

rectly described. Equally, at fixed density, the disorder dependence is roughly linear in Γ as predicted by Eq. (12).

The good quantitative agreement gives further weight to the main idea of our model in which there is a competition between the energy cost of a flipping spins, which increases with Γ , and the exchange gain of flipping spins, which increases with decreasing n_s , because of stronger exchange interactions at lower densities. The assumption that spin splitting in GaAs is driven primarily by exchange interactions, the Zeeman splitting being only a correction, is also validated.

From the experimentally determined critical magnetic field, B_{ss} , and the Landau level broadening, Γ , it is possible to calculate the critical density of states at the Fermi level, $D_{ss}(E_F) = (eB_{ss}/h)/\sqrt{\pi}\Gamma$, necessary to observe the paramagnetic to ferromagnetic phase transition. This is plotted in Fig. 10 versus the inverse exchange energy ($1/X_\infty$). Here, $1/X_\infty$ is calculated from the electron density using Eq. (11). The corresponding r_s values are also indicated on the top axis. The predicted dependence of $D_{ss}(E_F)$ on $1/X_\infty$, calculated using Eq. (12) is plotted in Fig. 10 for comparison. The agreement with theory is reasonably good, although there is some deviation at low electron density (high r_s). In these samples, spin splitting occurs too early, before the critical density of states is reached. This may be a correction due to the nonzero Zeeman energy. Clearly, in the limit of infinitely narrow Landau levels, our model breaks down, and the Zeeman energy is sufficient for the system to form a polarized ground state

in the highest Landau level, even in the absence of exchange interactions. The density of states for a 2D system in zero magnetic field is indicated by the dotted line in Fig. 10. The transition to a ferromagnetic state at zero magnetic field is predicted to occur for $r_s \sim 26$, corresponding to the divergence of (χ/χ_0) , in the calculations of Attacalite *et al.*¹⁶ However, the corresponding electronic density of $n_s \sim 5 \times 10^8 \text{cm}^{-2}$ is well below the density for the metal-insulator transition in GaAs.²⁷ At such densities, the system can no longer be described as a paramagnetic Fermi liquid, and hence no Stoner transition is expected at $B=0$ in GaAs, in agreement with experiment.

VI. CONCLUSION

In conclusion, we have performed a quantitative analysis of spin splitting in a series of GaAs heterojunctions and quantum wells. A simple model is developed which predicts that spin splitting should occur when the density of states at the Fermi level is large enough to stabilize a spin polarized ground state for the highest occupied Landau level. The density of states in magnetic field has been experimentally determined using low temperature Shubnikov de Haas measurements. The exchange strength has been estimated from the theoretical spin susceptibility of Ref. 16. Tilted-field measurements have been used to show that the measured spin susceptibility for our samples is in good agreement with this theory. In addition, the calculated exchange energy has been shown to be reasonable from an experimental estimation of the exchange-enhanced g factors. The predicted field for the appearance of spin splitting, B_{ss} , calculated in the limit of zero Zeeman energy and zero temperature, is in good quantitative agreement with the experimental data at mK temperature.

As the Zeeman energy plays no role in this model, the appearance of spin splitting is simply the manifestation of an itinerant quantum Hall ferromagnet in the highest occupied Landau level. This can also be thought of as a Stoner transition, since the only role of the magnetic field is to modify the density of states at the Fermi energy. The critical density of states, $D_{ss} = 1/X_\infty$, for the Stoner transition is larger than the two-dimensional density of states, $m^*/2\pi\hbar^2$, consistent with the absence of a Stoner transition at zero magnetic field.

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