Coupled spin-charge drift-diffusion equations for the Rashba model subject to an in-plane electric field

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Coupled spin-charge drift-diffusion equations are derived for a biased two-dimensional electron gas with weak Rashba spin-orbit interaction. The basic equations formally agree with recent results obtained for spin-orbit coupled small polarons. It is shown that effects of an in-plane electric field on a homogeneous spin system can completely be described by an associated in-plane magnetic field. Exploiting this analogy, we study among other things the electric-field equivalent of the Hanle effect.

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I. INTRODUCTION

The prospects of a new generation of electronic devices stimulate a renewed theoretical and experimental interest in the study of spin effects in semiconductors. A prerequisite for the design of semiconductor structures, whose function is based on electron spin, is a due understanding of spin dynamics and spin-polarized transport. In this field, the spin-orbit interaction (SOI) received particular interest since it allows purely electric manipulation of the electron spin. Many studies refer to a two-dimensional electron gas (2DEG), in which the Rashba SOI arises because of the quantum well asymmetry in the perpendicular direction. The SOI leads to a coupling between spin and charge degrees of freedom, which offers the possibility of controlling the spin polarization by an electric field.

The theoretical description of spin phenomena in semiconductors under the influence of SOI is based on appropriate transport equations. There are numerous approaches[1–6] that rely on a separation of drift-diffusion processes for spin and charge densities. Strictly speaking, such an approach is inappropriate when the SOI has to be accounted for. Spin-charge coupled drift-diffusion transport equations have been derived from firm microscopic models for extended states of a 2DEG[7–11] and for the hopping transport of small polarons.[12] Especially, the approach by Mishchenko *et al.*[7,8] for a 2DEG with Rashba SOI initiated many interesting studies.[13–16] Unfortunately, variants of these calculations suffer from an inconsistency because SOI contributions from the collision integral have been disregarded. The corrections lead to a cancellation of contributions that have been used in a previous work[13] to erroneously predict propagating coupled spin-charge waves.

The aim of the present paper is to derive spin-charge coupled drift-diffusion equations for a 2DEG with Rashba SOI that correct deficiencies of previous approaches and that describe the influence of an external in-plane electric field on spin polarization. With the help of universal macroscopic drift-diffusion equations, the spatial and temporal evolutions of coupled spin-charge disturbances as well as associated charge accumulation and magnetization are studied in semiconductor heterostructures with Rashba SOI. Our basic equations, which are derived for weak SOI, completely agree with results obtained for the hopping transport of small polarons.[12] The field-induced homogeneous spin accumulation as well as the charge-Hall current are treated. Furthermore, it is shown that for a homogeneous system, the effect of the electric field on spin polarization can be completely captured by a fictitious magnetic field. This analogy between the real applied electric field and an auxiliary magnetic field is used to predict a number of interesting electric-field effects on spin. We mention the decay of a spin polarization by a transverse electric field. Using this electric-field-driven Hanle effect by exchanging the in-plane magnetic field by an electric field in the measurement set up for the ordinary Hanle effect, it is possible to determine electron and spin lifetimes under steady-state conditions by varying the electric-field strength. Another application refers to the pseudo-charge-Hall effect,[17] which is induced by circular polarized light via the creation of a permanent spin magnetization.

II. KINETIC EQUATIONS

The effect of an in-plane electric field $\boldsymbol{E}$ on coupled spin-charge excitations of semiconducting electrons in an asymmetric quantum well can be described by a single-particle Hamiltonian

$$H_0 = \sum_{\boldsymbol{k},\lambda} a^{\dagger}_{\boldsymbol{k}\lambda}[\varepsilon_{\boldsymbol{k}} - \varepsilon_F]a_{\boldsymbol{k}\lambda} - \sum_{\boldsymbol{k},\lambda,\lambda'} (\hbar\boldsymbol{\omega}_{\boldsymbol{k}}\cdot\boldsymbol{\sigma}_{\lambda\lambda'})a^{\dagger}_{\boldsymbol{k}\lambda}a_{\boldsymbol{k}\lambda'} - ie\boldsymbol{E}\sum_{\boldsymbol{k},\lambda}\nabla_{\boldsymbol{\kappa}}a^{\dagger}_{\boldsymbol{k}-(\boldsymbol{\kappa}/2)\lambda}a_{\boldsymbol{k}+(\boldsymbol{\kappa}/2)\lambda}\bigg|_{\boldsymbol{\kappa}=\boldsymbol{0}} + u\sum_{\boldsymbol{k},\boldsymbol{k}'}\sum_{\lambda} a^{\dagger}_{\boldsymbol{k}\lambda}a_{\boldsymbol{k}'\lambda}, \tag{1}$$

which includes both the Rashba SOI and the short-range spin-independent elastic scattering on impurities. The Hamiltonian is expressed by creation $(a^{\dagger}_{\boldsymbol{k}\lambda})$ and annihilation $(a_{\boldsymbol{k}\lambda})$ operators that depend on the vector $\boldsymbol{k}=(k_x,k_y,0)$ and the spin index λ. We introduced the Fermi energy ε_F, the vector of Pauli matrices $\boldsymbol{\sigma}$, and the strength u of the short-range elastic impurity scattering, which is characterized by the momentum relaxation time τ. The energy $\varepsilon_{\boldsymbol{k}}$ of free electrons in the 2DEG and the coupling term of the Rashba SOI are given by

$$\varepsilon_{\boldsymbol{k}} = \frac{\hbar^2 \boldsymbol{k}^2}{2m}, \quad \boldsymbol{\omega}_{\boldsymbol{k}} = \frac{\hbar}{m}(\boldsymbol{K}\times\boldsymbol{k}), \quad \boldsymbol{K} = \frac{m\alpha}{\hbar^2}\boldsymbol{e}_z, \tag{2}$$

where m denotes the effective mass and α the strength of the SOI. The central quantity of our approach is the spin-density matrix $\hat{f}$, the components of which

$$f^{\lambda}_{\lambda'}(\boldsymbol{k},\boldsymbol{k}'|t) = \langle a^{\dagger}_{\boldsymbol{k}\lambda} a_{\boldsymbol{k}'\lambda'}\rangle_t, \tag{3}$$

satisfy kinetic equations, which are derived for the corresponding physical elements $f=\mathrm{Tr}\,\hat{f}$ and $\boldsymbol{f}=\mathrm{Tr}\,\boldsymbol{\sigma}\hat{f}$. The four components of the spin-density matrix depend on two wave vectors $(\boldsymbol{k}+\boldsymbol{k}')/2\to\boldsymbol{k}$ and $(\boldsymbol{k}-\boldsymbol{k}')\to\boldsymbol{\kappa}$, which allow the description of inhomogeneous charge and spin distributions. Relying on the Born approximation for the treatment of elastic impurity scattering and restricting to lowest-order corrections of the SOI to the collision integral, we obtain the following Laplace-transformed kinetic equations:[18]

$$sf - \frac{i\hbar}{m}(\boldsymbol{\kappa}\cdot\boldsymbol{k})f - \frac{i\hbar}{m}\boldsymbol{K}(\boldsymbol{f}\times\boldsymbol{\kappa}) + \frac{e\boldsymbol{E}}{\hbar}\nabla_{\boldsymbol{k}} f = \frac{1}{\tau}(\bar{f}-f) + f_0, \tag{4}$$

$$\boldsymbol{s f} + 2(\boldsymbol{\omega}_{\boldsymbol{k}}\times\boldsymbol{f}) - \frac{i\hbar}{m}(\boldsymbol{\kappa}\cdot\boldsymbol{k})\boldsymbol{f} + \frac{i\hbar}{m}(\boldsymbol{K}\times\boldsymbol{\kappa})f + \frac{e\boldsymbol{E}}{\hbar}\nabla_{\boldsymbol{k}}\boldsymbol{f} = \frac{1}{\tau}(\bar{\boldsymbol{f}}-\boldsymbol{f}) + \frac{1}{\tau}\frac{\partial}{\partial\varepsilon_{\boldsymbol{k}}}\overline{f\hbar\boldsymbol{\omega}_{\boldsymbol{k}}} - \frac{\hbar\boldsymbol{\omega}_{\boldsymbol{k}}}{\tau}\frac{\partial}{\partial\varepsilon_{\boldsymbol{k}}}\bar{f} + \boldsymbol{f}_0, \tag{5}$$

with the initial charge and spin distribution f_0 and $\boldsymbol{f}_0$, respectively. An integration over the polar angle φ of the vector $\boldsymbol{k}$ is indicated by a cross line over respective quantities, and s denotes the Laplace variable that refers to the time t. The second and third terms on the right-hand side of Eq. (5) stem from spin contributions of the collision integral, which have to be taken into account to guarantee that the spin system correctly approaches the state of thermodynamic equilibrium. The solution of the coupled integrodifferential equations (4) and (5) is searched for in the long-wavelength and low-frequency regime.

To illustrate our approach, let us first treat the evolution of charge-density disturbances at zero SOI. As the inelastic scattering time τ_ε is usually much larger than τ, the quasimomenta thermalize already at the time scale $\tau_\varepsilon > t > \tau$ ($s\tau \ll 1$). During this period, the density matrix $(f,\boldsymbol{f})$ approaches its mean value $(\bar{f},\bar{\boldsymbol{f}})$ with respect to the angle φ. In the following stage of the evolution $t>\tau_\varepsilon$, which lasts until a characteristic diffusion time τ_d, the carrier density locally approaches the equilibrium distribution. Consequently, the behavior of particles in this time interval can be described by the Fermi function $n(\varepsilon_{\boldsymbol{k}})$ with a Fermi energy that depends on spatial coordinates $\boldsymbol{r}$ and time t: $f(\varepsilon_{\boldsymbol{k}},\boldsymbol{r}|t)=n[\varepsilon_{\boldsymbol{k}}-\varepsilon_F(\boldsymbol{r},t)]$. In this evolution period, the energy is already thermalized although both the charge and spin densities still remain inhomogeneous. For weak perturbations $\varepsilon_F(\boldsymbol{r},t)=\varepsilon_F+\Delta\varepsilon_F(\boldsymbol{r},t)$, we obtain for the density fluctuation $\delta f(\varepsilon_{\boldsymbol{k}},\boldsymbol{r}|t)=-\Delta\varepsilon_F(\boldsymbol{r},t)dn(\varepsilon_{\boldsymbol{k}}-\varepsilon_F)/d\varepsilon_{\boldsymbol{k}}$. This result brings us to the separation ansatz

$$\bar{f}(\varepsilon,\boldsymbol{\kappa}|s) = -F(\boldsymbol{\kappa}|s)\frac{n'(\varepsilon)}{dn/d\varepsilon_F} \tag{6}$$

for the new unknown function $F(\boldsymbol{\kappa}|s)$, where $n=\int d\varepsilon\rho(\varepsilon)n(\varepsilon)$ with $\rho(\varepsilon)$ being the density of states of the 2DEG. For brevity, we write ε instead of $\varepsilon_{\boldsymbol{k}}$ and use a prime to indicate a derivative with respect to ε. It is in line with this discussion and Eq. (6) to replace the drift term accordingly

$$\frac{e}{\hbar}E_x\frac{\partial}{\partial k_x}f \to eE_x\frac{\hbar k}{m}\frac{n''}{n'}\cos(\varphi)\bar{f}. \tag{7}$$

Adopting these approximations, which express the basic understanding of the drift-diffusion approach, Eq. (4) is easily solved under the condition of vanishing SOI ($\boldsymbol{K}=\boldsymbol{0}$). A spectral drift-diffusion equation is obtained by expanding the solution of Eq. (4) with respect to $\boldsymbol{\kappa}$ and by integrating over the angle φ

$$\left[s + D(k)\kappa^2 + i\mu\varepsilon\frac{n''}{n'}\boldsymbol{E}\cdot\boldsymbol{\kappa}\right]\bar{f}(\varepsilon,\boldsymbol{\kappa}|s) = f_0, \tag{8}$$

where the diffusion coefficient and the mobility are given by $D(k)=(\hbar k)^2\tau/(2m^2)$ and $\mu=e\tau/m$, respectively. The final integration over the energy ε leads to the well-known drift-diffusion equation

$$[s + D\kappa^2 - i\mu\boldsymbol{E}\cdot\boldsymbol{\kappa}]\bar{f}(\boldsymbol{\kappa}|s) = f_0, \tag{9}$$

for the charge density $\bar{f}(\boldsymbol{\kappa}|s)=\int d\varepsilon\rho(\varepsilon)\bar{f}(\varepsilon,\boldsymbol{\kappa}|s)$. The relationship between the diffusion coefficient D and the mobility μ is given by the Einstein relation $\mu=(eD/n)dn/d\varepsilon_F$, which is applicable both for Fermi and Boltzmann statistics.

Within the framework of the drift-diffusion approach, a similar approximation can be used for the spin components of the density matrix when the Rashba SOI is weak ($\Omega=\omega_k\tau\ll 1$, which is accessible by tuning the SOI coupling constant α via the shape of the confinement potential). In this case, the spin-relaxation time τ_s is large so that we can focus on the time hierarchy $\tau<\tau_\varepsilon<\tau_s<\tau_d$. Under the condition $t>\tau_\varepsilon$ but t/τ_s arbitrary, a nonequilibrium spin polarization exists on the background of thermalized carrier energies. Therefore, Eqs. (6) and (7) can be used also for the spin contributions in the kinetic equations (4) and (5). Exploiting these approximations, the following set of linear equations is obtained for the components of the spin-density matrix:

$$\sigma f + i\Omega(q_x f_y - q_y f_x) = R - \frac{2eE\tau}{\hbar k}\varepsilon\frac{n''}{n'}\cos(\varphi)\bar{f},$$

$$\sigma f_x + 2\Omega\cos(\varphi)f_z - i\Omega q_y f = R_x + \frac{2}{\gamma}\varepsilon\frac{n''}{n'}\sin(\varphi)\bar{f} - \frac{2eE\tau}{\hbar k}\varepsilon\frac{n''}{n'}\cos(\varphi)\bar{f}_x,$$

$$\sigma f_y + 2\Omega \sin(\varphi) f_z + i\Omega q_x f = R_y - \frac{2}{\gamma}\varepsilon\frac{n''}{n'}\cos(\varphi)\bar{f} - \frac{2eE\tau}{\hbar k}\varepsilon\frac{n''}{n'}\cos(\varphi)\bar{f}_y,$$

$$\sigma f_z - 2\Omega[\cos(\varphi) f_x + \sin(\varphi) f_y] = R_z - \frac{2eE\tau}{\hbar k}\varepsilon\frac{n''}{n'}\cos(\varphi)\bar{f}_z, \qquad (10)$$

with $q_{x,y}=\kappa_{x,y}/k$, $\gamma=k/K$, and

$$\sigma = \sigma_0 - i\gamma\Omega(q_x \cos\varphi + q_y \sin\varphi), \quad \sigma_0 = s\tau + 1. \qquad (11)$$

It is assumed that the in-plane electric field $\boldsymbol{E}$ is oriented along the x axis. Restricting to lowest-order contributions in $\boldsymbol{\kappa}$ and $\boldsymbol{E}$, we obtain

$$R_x = \bar{f}_x + \tau f_{x0} - \frac{i\hbar K\tau}{m\sigma_0^2}\frac{(\varepsilon n')'}{n'}\kappa_y \bar{f}, \quad R = \bar{f} + \tau f_0,$$
$$R_z = \bar{f}_z + \tau f_{z0}, \qquad (12)$$

$$R_y = \bar{f}_y + \tau f_{x0} + \frac{i\hbar K\tau}{m\sigma_0^2}\frac{(\varepsilon n')'}{n'}\kappa_x \bar{f} - \frac{\hbar K\tau}{m\sigma_0}eE\frac{(\varepsilon n'')'}{n'}\bar{f}. \qquad (13)$$

It is straightforward but cumbersome to solve the equations for f, $\boldsymbol{f}$, to expand the solution with respect to κ, and to calculate the final integral over the angle φ. What we obtain by this procedure are spectral drift-diffusion equations for coupled spin-charge excitations. After integrating over the remaining energy ε, we get our final result

$$\left[\frac{\partial}{\partial t} - i(\boldsymbol{\kappa}\cdot\mu\boldsymbol{E}) + D\kappa^2\right]F - \frac{i\hbar}{m}\boldsymbol{\kappa}\cdot[\boldsymbol{K}\times\boldsymbol{F}] + \frac{2i\hbar\mu}{e}(\boldsymbol{\kappa}\cdot[\boldsymbol{K}\times\mu\boldsymbol{E}])(\boldsymbol{K}\cdot\boldsymbol{F}) = 0, \qquad (14)$$

$$\left[\frac{\partial}{\partial t} - i(\boldsymbol{\kappa}\cdot\mu\boldsymbol{E}) + D\kappa^2 + \frac{\hat{A}}{\tau_s}\right]\boldsymbol{F} + 4D\left\{\left[\boldsymbol{K}\times\left(i\boldsymbol{\kappa} + \frac{\mu}{2D}\boldsymbol{E}\right)\right]\times\boldsymbol{F}\right\} + \left\{\frac{2i\hbar\mu}{e}(\boldsymbol{\kappa}\cdot[\boldsymbol{K}\times\mu\boldsymbol{E}])\boldsymbol{K} - \frac{2\hbar\mu}{e\tau_s}\left[\boldsymbol{K}\times\left(i\boldsymbol{\kappa} + \frac{\mu}{2D}\boldsymbol{E}\right)\right]\right\}F = 0, \qquad (15)$$

with $A_{xx}=A_{yy}=1$, $A_{zz}=2$, and the spin-scattering time calculated from $1/\tau_s=4DK^2$. In Eq. (15), the electric field is accounted for via the quasichemical potential ($i\boldsymbol{\kappa}\to i\boldsymbol{\kappa}+\mu\boldsymbol{E}/2D$). These coupled spin-charge drift-diffusion equations are valid for weak SOI ($\Omega \ll 1$ so that $\tau/\tau_s \ll 1$). What is interesting is that Eqs. (14) and (15) formally agree with results which were recently derived by a different approach for the hopping transport of small polarons.[12] Spin effects in the latter system exclusively occur in the weak SOI regime as the lattice constant is much smaller than typical values of K^{-1}. Summarizing this observation, we point out that the drift-diffusion equations have a universal character for the Rashba model with weak SOI.

III. RESULTS AND DISCUSSION

The aforementioned analogy with the hopping transport provides us a recipe to transfer results recently obtained for small polarons[12] in a straightforward manner to spin effects of extended electronic states. We shall not consider all examples and not repeat all calculations already presented in Ref. 12, but restrict ourselves to some additional conclusions.

A. Expression for the spin current

To begin with, let us express our main result, namely, the drift-diffusion equations (14) and (15) in spatial coordinates. For the charge density, we obtain the continuity equation

$$\frac{\partial F(\boldsymbol{r},t)}{\partial t} + \operatorname{div}\boldsymbol{j}(\boldsymbol{r},t) = 0, \qquad (16)$$

with a particle current

$$\boldsymbol{j} = (\mu\boldsymbol{E} - D\nabla_r)F + \frac{\hbar}{m}[\boldsymbol{K}\times\boldsymbol{F}] - \frac{2\hbar\mu^2}{e}K[\boldsymbol{K}\times\boldsymbol{E}]F_z \qquad (17)$$

that includes both charge and spin components. Besides the charge-Hall current

$$\boldsymbol{j}_H = -2\hbar\mu^2 K[\boldsymbol{K}\times\boldsymbol{E}]F_z, \qquad (18)$$

which arises from a given out-of-plane spin polarization F_z, there appears another spin-related term that is responsible for the spin-galvanic effect. As the spin is not conserved, the equation for the spin components F_α of the density matrix cannot be written in the form of a continuity equation. Rather, we obtain from Eq. (15)

$$\frac{\partial F_\alpha}{\partial t} + \frac{A_\alpha}{\tau_s}F_\alpha + 2\mu([\boldsymbol{K}\times\boldsymbol{E}]\times\boldsymbol{F})_\alpha - \frac{\hbar\mu}{\tau_s}\frac{n'}{n}[\boldsymbol{K}\times\boldsymbol{E}]_\alpha F + \frac{\partial J_{i\alpha}}{\partial r_i} = 0, \qquad (19)$$

where the spin current $J_{i\alpha}$ is given by

$$\boldsymbol{J}_\alpha = (\mu\boldsymbol{E} - D\nabla_r)F_\alpha + \delta\boldsymbol{J}_\alpha, \qquad (20)$$

with the spin components

$$\delta\boldsymbol{J}_0 = \frac{\hbar}{m}(\boldsymbol{K}\times\boldsymbol{F}) - \frac{2\hbar K\tau}{m}(\boldsymbol{K}\times\mu\boldsymbol{E})F_z,$$
$$\delta\boldsymbol{J}_z = \frac{1}{K\tau_s}\left[\boldsymbol{F} - \frac{\hbar\tau}{2Dm}(\boldsymbol{K}\times\mu\boldsymbol{E})F\right],$$
$$\delta\boldsymbol{J}_x = -\frac{1}{K\tau_s}\left(F_z\boldsymbol{e}_x + \frac{\hbar K^2\tau}{m}F\boldsymbol{e}_y\right),$$
$$\delta\boldsymbol{J}_y = \frac{1}{K\tau_s}\left(\frac{\hbar K^2\tau}{m}F\boldsymbol{e}_x - F_z\boldsymbol{e}_y\right). \qquad (21)$$

Studies of the spin current received a great deal of recent interest in the literature.[18–24] This interesting discussion is confronted with a serious problem, namely, that the spin is not conserved. As a consequence, different definitions of the spin current have been put forward in the literature. According to Eq. (21), we obtain for the spin-Hall current $J_y^z = -2\hbar K^2\mu^2 EF/e$, which is neither universal[25] nor in line with the result derived from a more physically motivated definition of the spin-Hall current.[18,19] In addition, there is a component of the spin current that is completely independent of the electric field $[J_y^x = -(\hbar K/m)(\tau/\tau_s)F]$. This astonishing result is replaced in an alternative approach[18] by a spin current that is related to the initial variation of the spin accumulation and that disappears when the latter reaches its steady-state value. As the concept of the spin current is not well founded, it seems to be expedient to avoid its introduction.

B. Electric-field-induced spin effects in a homogeneous electron gas

As another application of the kinetic equations (14) and (15), we treat the field-induced homogeneous ($\boldsymbol{\kappa}=\mathbf{0}$) spin accumulation. From Eq. (15), we obtain for the steady-state field-mediated magnetic moment

$$\bar{\boldsymbol{f}} = \hbar\mu(\boldsymbol{K}\times\boldsymbol{E})n'. \tag{22}$$

This result expresses the well-known magnetoelectric effect that was predicted by Edelstein[26] many years ago. For its derivation, it was essential to account for spin contributions on the right-hand side of Eq. (5), the origin of which is the collision integral. If we would neglect these corrections, a term of the form $i\hbar K\kappa_x\bar{f}/m$ remained uncompensated in Eq. (15), which was used in a recent paper[13] to predict coupled spin-charge waves. Introducing an electric field via the quasichemical potential,[18] this term would lead to a magnetic moment $\bar{\boldsymbol{f}}\sim 1/K$, in complete disagreement with well established results.[26]

Furthermore, it is worth noting that the effect of an electric field on the spin polarization can completely be simulated by an appropriate in-plane magnetic field of the form

$$\boldsymbol{H}_{\text{eff}} = \frac{\hbar\mu}{\mu_B}[\boldsymbol{K}\times\boldsymbol{E}]. \tag{23}$$

Replacing the electric field by this equivalent magnetic field, Eq. (19) is written as

$$\frac{\partial\boldsymbol{F}}{\partial t} + \frac{\hat{A}}{\tau_s}\boldsymbol{F} + \frac{2\mu_B}{\hbar}[\boldsymbol{H}_{\text{eff}}\times\boldsymbol{F}] - \frac{1}{\tau_s}\frac{\chi\boldsymbol{H}_{\text{eff}}}{\mu_B} = \mathbf{0}, \tag{24}$$

with $\chi=\mu_B^2 n'$ being the magnetic susceptibility. The term $\chi\boldsymbol{H}_{\text{eff}}$ is responsible for the spin accumulation, whereas the vector product $\boldsymbol{H}_{\text{eff}}\times\boldsymbol{F}$ leads to spin precession around the effective magnetic field $\boldsymbol{H}_{\text{eff}}$. This close relationship between spin polarization due to an electric field and its description by an associated in-plane magnetic field can be used for the derivation and interpretation of electric-field effects on spin. As an example, we mention the rotation of an initial perpendicular homogeneous spin polarization $F_z(t=0)=F_{z0}$ into the plane of the 2DEG due to Larmor precession. For a constant electric field oriented along the x axis, we obtain from Eq. (24)

$$F_x(t) = -\frac{2\mu_B H_{\text{eff}}}{\hbar}\frac{\sin(\omega_s t)}{\omega_s}\exp\left(-\frac{3t}{2\tau_s}\right)F_{z0},$$
$$\omega_s = \sqrt{\left(\frac{2\mu_B H_{\text{eff}}}{\hbar}\right)^2 - \frac{1}{(2\tau_s)^2}}, \tag{25}$$

with $H_{\text{eff}}=\hbar\mu KE/\mu_B$. This solution demonstrates that sufficiently high electric fields lead to an in-plane spin polarization that oscillates with the frequency $\omega_s\approx 2\mu_B H_{\text{eff}}/\hbar = 2\mu EK$.

To further exploit the analogy between electric and magnetic field effects, let us treat the optical generation and recombination of a steady-state spin polarization under the additional influence of a real in-plane magnetic field $\boldsymbol{H}$ oriented along the y axis. In this case, Eq. (24) takes the form

$$\frac{\partial\delta\boldsymbol{F}}{\partial t} + \frac{\hat{A}}{\tau_s}\delta\boldsymbol{F} + \frac{2\mu_B}{\hbar}[\boldsymbol{B}\times\delta\boldsymbol{F}] = \boldsymbol{G} - \frac{\delta\boldsymbol{F}}{\tau_0}, \quad \delta\boldsymbol{F} = \boldsymbol{F} - \frac{\chi\boldsymbol{B}}{\mu_B}, \tag{26}$$

where $\boldsymbol{B}=\boldsymbol{H}+\boldsymbol{H}_{\text{eff}}$. The vector $\boldsymbol{G}$ describes the optical out-of-plane spin generation and τ_0 is the relaxation time of photogenerated electrons. The steady-state solution of Eq. (26) is easily obtained

$$\delta F_z = \frac{\tau_\perp G_z}{1+\omega_c^2\tau_\parallel\tau_\perp}, \quad \frac{1}{\tau_\parallel} = \frac{1}{\tau_s} + \frac{1}{\tau_0}, \quad \frac{1}{\tau_\perp} = \frac{2}{\tau_s} + \frac{1}{\tau_0}, \tag{27}$$

with $\omega_c=2\mu_B B/\hbar$. At zero electric field, the solution describes the depolarization of spin (and hence, the degree of circular polarization of the luminescence) by a transverse magnetic field, which is known as the Hanle effect.[27] Combining the measurement of the zero-field spin generation G_z with the magnetic-field dependence in Eq. (27) (Hanle effect), both the electron lifetime and the spin-relaxation time τ_s can be determined.[28–33] According to Eq. (27), an effect of the same kind exists also at zero magnetic field ($\boldsymbol{H}=\mathbf{0}$) due to an in-plane electric field. To describe this effect, it is only necessary to replace the Larmor frequency $2\mu_B B/\hbar$ by $\omega_c = 2\mu EK$ in Eq. (27). This Hanle effect driven by a pure in-plane electric field can likewise be used to measure lifetimes of charge and spin excitations. A realistic estimate of the electric-field-mediated effective magnetic field leads to the value $H_{eff}\approx 1$ kOe,[34] the magnitude of which is large enough to induce measurable changes in the Hanle curves.[34] The experimental results qualitatively agree with the theoretical prediction of the effect.

Concerning the charge transport, we obtain the result that a circular polarized light illumination induces a pseudo-Hall effect[17] in the absence of any external magnetic field. A quantitative description of this Hall contribution is provided by Eqs. (18) and (27). Another application of the electric-magnetic field correspondence refers to a modification of recent experiments, in which the optically induced spin-galvanic effect was measured.[35] Instead of using an external magnetic field to achieve an in-plane spin polarization necessary for the occurrence of the effect, one can likewise apply an in-plane electric field.

C. Field-induced spin waves

Finally, let us treat spin waves that exist in a stripe of a 2DEG oriented parallel to the electric field (and the x axis). Taking into account $\tau/\tau_s \ll 1$ and restricting ourselves to long wavelengths ($D\kappa_y^2\tau_s \ll 1$), we obtain the dispersion relation

$$\omega_{1,2} = \frac{3}{2\tau_s}\left[i \pm \sqrt{\frac{\tau_s}{\tau_E} - 1}\right] + D\kappa_y^2\left[i \pm \frac{2}{3\sqrt{\tau_s/\tau_E - 1}}\right], \tag{28}$$

in which a rate $1/\tau_E = (\mu E)^2/(9D)$ appears, which is associated with the electric field. Field-induced damped oscillations arise at sufficiently high electric field strengths ($\tau_s/\tau_E > 1$). For typical parameters $\mu \approx 10^6$ cm^2/V s, $\tau_s \approx 40$ ps, $\tau = 0.5$ ps, and $n = 10^{11}$/cm^2, we obtain the condition $E > 3.25$ V/cm, which gives rise to an appreciable current density in the 2DEG. A much stronger electric field ($\tau_s/\tau_E \gg 1$) drives spin-charge coupled oscillations with the constant field-dependent frequency $\omega = |\mu E|/(2\sqrt{D\tau_s})$. In contrast to the well-known space-charge waves of free and trapped electrons, this massive mode with the frequency $\omega = K\mu E$ is independent of the propagation vector $\boldsymbol{\kappa}$.

IV. SUMMARY

Based on the density-matrix approach, spin-charge coupled drift-diffusion equations were derived for extended electronic states in a 2DEG with weak SOI. The final basic equations agree with results that were recently obtained for the hopping transport of spin-polarized polarons.[12] Due to this correspondence, results on spin transport obtained for localized and extended states are mutually applicable to each other. In the course of the derivation, it was found that a consistent treatment of spin-charge coupling requires a careful consideration of spin-orbit contributions to the collision integral, which give rise to a tricky cancellation in transport equations. Disregarding these corrections is the source of fatal mistakes that plague former approaches.

Particular emphasis was put on the effect of an electric field on the spin polarization of a homogeneous 2DEG. It was shown that the electric field can be replaced by a fictitious in-plane magnetic field in the drift-diffusion equations for the spin components. This interpretation of the equations reveals a number of interesting similarities between spin effects induced by electric or magnetic fields. From an experimental point of view, most attractive seems to be the electric-field equivalent of the Hanle effect, which provides another possibility to measure lifetimes of spin and charge excitations by manipulating exclusively an in-plane external electric field.

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