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Mid-infrared pump-related electric-field domains in GaAs/(Al,Ga)As quantum-cascade structures for terahertz lasing without population inversion

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We investigate the effect of mid-infrared (MIR) pumping on the transport properties of GaAs/(Al,Ga)As terahertz (THz) quantum lasers (TQLs), which rely on quantum coherence effects of intersubband transitions. Aiming at THz lasing at elevated temperatures, we extend the concept of THz gain with and without population inversion of a single, MIR-pumped, electrically driven THz stage proposed by Waldmuller et al. [Phys. Rev. Lett. 99, 117401 (2007)] to an entire TQL. However, experiments using a CO₂ as well as a free-electron laser and numerical simulations show that this resonant MIR pumping causes a negative differential conductivity (NDC) in addition to the NDC caused by sequential tunneling. Lasing of these TQLs is prevented by the formation of electric-field domains below the resonance field strength for gain of each single THz stage.

I. INTRODUCTION

Remarkable success has been achieved in the development of terahertz (THz) quantum-cascade lasers (QCLs) for the past decade, ranging from understanding of THz lasing relying on population inversion of intersubband states to the demonstration of numerous applications (see, e.g., Ref. 1). However, the operation of THz QCLs usually requires cryogenic cooling.¹⁻⁴ In order to increase the population inversion between the upper and lower laser levels and, therefore, the operating temperature of THz QCLs, two design principles have been studied: fast depopulation of the lower THz laser state⁵ and enhanced injection into the upper THz laser state,⁶⁻¹⁰ which are both based on resonant longitudinal optical (LO) phonon scattering. However, the maximum operating temperatures T_{max} achieved so far are 186 K for lasing at about 4 THz (Ref. 3) and 180 K for about 2 THz,⁶ although at least 240 K would be necessary for Peltier cooling. Recently, we have demonstrated a cryogen-free operation of a THz QCL using a Stirling cooler.¹¹

Experiments show that the population inversion for most THz QCLs seems to be limited by the condition E_{0} ≤ k_B T_{max}, where E_{0} denotes the lasing energy and k_B Boltzmann’s constant. This inequality cannot be strictly derived, but it may be illustrated by plausible arguments for QCL states, which are not very strongly localized.¹⁻³,⁶ Therefore, concepts for lasing without inversion are the crucial challenge for room temperature devices. Two approaches of second-order processes and, therefore, not very effective lasing mechanisms as the first-order population inversion process are subject of investigations: First, lasing due to intracavity difference frequency generation of two mid-infrared (MIR) QCLs demonstrated by Belkin et al.¹²,¹³ at 300 K. Second, lasing without population inversion, which is based on quantum coherence due to quantum interference between the microscopic polarizations of optical transitions in three-level V or Λ systems, demonstrated for atomic transitions¹⁴⁻¹⁹ and investigated for MIR intersubband transitions by, e.g., Müller et al.²⁰ and Frogly et al.²¹

Recently, Waldmuller et al.²² extended the three-level scheme to a closed loop, four-level, MIR-pumped, electrically driven THz emitting stage. At the resonance field strength F = F_0, this four-level stage is expected to convert the MIR pump into THz photons more effectively than a three-level unit, because the energy of the MIR pump transition is equal to that of the MIR recovery transition (E_{pump} = E_{rec}) and the energy of the emitted THz photons is supplied by the driving electric field (in contrast to V or Λ systems, where the THz energy is supplied by the MIR pump field).²²,²³ Furthermore, at resonance, a four-level system inherently conserves the total wavevector of all MIR transitions, which allows for a large variability of the pump geometry.²³ To form a compact THz laser operating at room temperature, the MIR pump radiation may be generated by a MIR QCL, which can be optically coupled in close proximity to the THz stages, similar to the arrangements used in studies of coupled multisection QCLs.²⁴

Figure 1 shows the subband structure of one period of a MIR-pumped, electrically driven entire THz quantum laser (TQL). Each period is formed by a THz stage coupled to an injector. The MIR pump ground state is denoted MIR_{pump}
pumping, which is also necessary for achieving gain. For the MIR pumping, we use a CO₂ laser and the free-electron laser facility FELBE at the Helmholtz-Zentrum Dresden-Rossendorf. We compare the experimental results with simulations.

II. SUBBAND STRUCTURE OF A MIR-PUMPED, ELECTRICALLY DRIVEN TQL

Figure 1 shows the subband structure of a TQL, which is designed for a pump wavelength of 10.6 μm (117 meV). In this TQL, the fast dephasing of the final recovery state of the THz stage into the next injector as well as the injection of electrons into the pump ground state is enhanced by resonant LO phonon emission.

The subband structure shown in Fig. 1 was simulated self-consistently by taking into account the Schrödinger and Poisson equations within the framework of a system of linear transport equations according to our model published in Refs. 25–27. The intersubband transition rates \( T_{ij} \) are approximated by

\[
T_{ij} = |D_{ij}|^2 \left[ \tau(E_{ij}) + \frac{\pi e^2}{\epsilon_0} \sum_k e_k \delta_{ij} L_{ij}(\epsilon_k) \right],
\]

where \( i \) and \( j \) denote subband indices, \( D_{ij} \) the dipole moment, and \( E_{ij} \) the transition energy. \( \tau(E_{ij}) \) approximates all averaged energy-dependent scattering events of the electrons on LO phonons, ionized impurities, interface roughness, and other electrons, which are assumed to be independent of the wavevector. Furthermore, \( e \) denotes the elementary charge, \( \epsilon \) the effective dielectric constant of the TQL cavity \((\sqrt{\epsilon} = 3.61)\), \( \epsilon_0 \) the permittivity of the vacuum, \( \omega \) the photon angular frequency, \( S_i \) the photon density of the cavity mode \( k \), \( L_{ij} \) takes into account the line broadening of the \( ij \) transition. Assuming a broadening of 1 meV due to interface roughness, we obtain a total line broadening for intersubband transitions of about 3 meV. For the simulation of the subband structure, gain, and current-voltage characteristics, we extend our previous model²⁵–²⁷ by including also the MIR pump and recovery processes by the photon densities \( S \) of the corresponding terms in the sum of Eq. (1). Furthermore, we include, in our self-consistent calculation, also the gain-quality loss condition for the balance of the MIR photon emission and absorption, but, for simplicity, neglect such a balance equation for the emitted THz photons. For the free-carrier waveguide absorption coefficient of \( p \)-polarized MIR light propagating parallel to the plane of the TQL layers, we estimate \( \alpha_{\text{MIR}} \approx 10 \text{ cm}^{-1} \) by using an effective medium approach for the dielectric function of MIR QCLs.²⁸ The reabsorption of MIR and THz photons due to intersubband excitations is directly taken into account in our calculations of the gain \( G \), because \( G(\omega) \approx \sum_{ij} \delta_{ij} (n_i - n_j) |D_{ij}|^2 L_{ij}(\omega) / C_0 C_1 \).

Figure 2 shows the calculated gain characteristic of the TQL for population inversion as a function of the driving field strength \( F_{\text{pert}} \) for a single period and the frequency of the emitted THz photons \( \nu_\text{THz} = \omega_\text{THz} / (2\pi) \).
power density corresponds roughly to a pump intensity of 2.5 MW/cm² and approximately agrees with the value assumed by Waldmueller et al. The calculated gain characteristic exhibits a local maximum with a value of GTHz ≈ 30 cm⁻¹ at about 4 THz for the resonance field strength for gain F₀ = F_per ≈ −21 kV/cm. Furthermore, our calculations confirm that, at the resonance field strength, (i) the energy of the MIR recovery transitions agrees with that of the MIR pump transition (CO₂ laser, 117 meV), (ii) the THz emission corresponds to an energy of 17.6 meV, (iii) the dipole moment for the allowed MIR pump transition (D_MIR_pump,allow = 0.23 nm) is much larger than that of the forbidden MIR pump process (D_MIR_pump,forb = 0.063 nm), (iv) the dipole moment between both THz laser states is the largest one (D_MIR_pump,allow = 2.4 nm), and (v) the dipole moment of the allowed MIR recovery process (D_MIR_rec,allow = 0.97 nm) is clearly larger than one of the forbidden recovery process (D_MIR_rec,forb = 0.33 nm). These calculated dipole moments are comparable to those values given by Waldmueller et al. for a single THz stage, so that also THz gain due to population inversion as well as to GWI is expected for a single period (e.g., a coupled THz stage-injector unit and not just an uncoupled single-period THz stage) for resonant MIR pumping at the resonance field strength F = F₀.

III. SAMPLES AND EXPERIMENTAL DETAILS

Similar to conventional THz QCLs, the studied TQLs contains 150 periods. The barriers consist of Al₀.₅Ga₀.₅As layers, and the wells in the injectors are formed by GaAs layers, while, for the THz wells, we use Al₀.₁₅Ga₀.₈₅As layers. The TQLs were grown by molecular-beam epitaxy. The nominal sheet carrier concentration N₀ was 3 × 10¹¹ cm⁻² per period. We use THz single-plasmon waveguides, i.e., 80 nm GaAs with Si doping of 5 × 10¹⁸ cm⁻³ as the top contact layer and 700 nm GaAs with Si doping of 2 × 10¹⁸ cm⁻³ as the bottom contact layer. The layer thicknesses and doping densities have been confirmed by x-ray diffraction and capacitance-voltage measurements, respectively. The laser ridges have an area of about 0.1 × 1 mm² and a thickness of about 10 μm.

The time-integrated characteristics of the current density versus electric-field strength (J-F_av) are measured for TQLs driven by a constant voltage at low temperatures (10 K). For pulsed MIR pumping, we use a CO₂ laser (Mini TEA CO₂ laser, Edinburgh Instruments) and the free-electron laser facility FELBE at the Helmholtz-Zentrum Dresden-Rossendorf. The wavelength of the CO₂ laser pulses can be varied between 9.2 and 12.1 μm, a pulse width of 3 ps, a repetition rate of 13 MHz, and a maximum average power on the sample of about 200 mW. In both cases, the unfocused MIR pump light is coupled into the TQLs through 45° side facets.

IV. EXPERIMENTAL CURRENT-VOLTAGE CHARACTERISTICS

We discuss the transport properties in terms of the current density as a function of the average field strength F_av, which is determined by the applied voltage V and the total length of the TQL according to F_av = V/(N_per × d_per), where N_per and d_per denote the number and length of the period, respectively. Figure 3 shows the J-F_av characteristics of the TQL without MIR pumping (dark current) and with MIR pulsed pumping using the CO₂ laser at 10.5 μm with a p- or s-polarized beam. The dark J-F_av characteristic exhibits a pronounced multistable behavior for field strengths between F_av ≈ F_d⁻ ≈ −13 kV/cm and F ≈ F_d⁺ ≈ −28 kV/cm and a hysteresis between an up sweep of |F_av| (high-current branch) and the down sweep (low-current branch). In an up sweep, we observed 136 current jumps in the J-F_av characteristic, where adjacent current maxima are separated by integers of the field strength difference ΔF_av = m × 0.084 kV/cm. These jumps indicate the formation of electric-field domains (EFDs) between F_d⁻ and F_d⁺. We have observed that

FIG. 2. (Color online) Calculated gain due to population inversion between the two THz states for a single period of the TQL for pumping with p-polarized light of 10.6 μm using a homogeneous power density of 10 MW/cm³ within the TQL cavity. The circle indicates the expected preferred region for THz lasing based on population inversion as well as GWI.

FIG. 3. (Color online) Measured time-integrated J-F_av characteristics of the constant-voltage-biased TQL without pumping (line, up and down sweep) and with pulsed pumping with the CO₂ laser at 10.5 μm using p- (circles, up sweep) and s-polarized beams (triangles, up sweep).
the jumps occur over up to three periods \((m = 1, 2, 3)\) in a typical up sweep.

For pulsed pumping with the CO\(_2\) laser, the up sweep of the time-integrated \(J-F_{\text{av}}\) characteristic of the TQL driven by a constant voltage follows for low values of \(|F_{\text{av}}|\) the high-current branch of the dark \(J-F_{\text{av}}\) characteristics. However, it switches at higher field strengths \(|F_{\text{av}}| \geq 17\, \text{kV/cm}\) from the high-current to the low-current branch of the dark case (cf. circles in Fig. 3). For \(s\)-polarized pumping, both the up-sweep and down-sweep characteristics almost coincide with the low-current branch of the curve for field values between \(-14\) and \(-20\, \text{kV/cm}\) (cf. triangles in Fig. 3). A pronounced change of this behavior was not observed for a variation of the pump wavelength between 9.2 and 11.2 \(\mu\text{m}\). In contrast to MIR pumping with the CO\(_2\) laser, we have not observed any effect of MIR pumping on the time-integrated \(J-F_{\text{av}}\) characteristic for pumping with the FELBE beam.

V. TRANSPORT MODEL FOR TQLs IN THE PRESENCE OF ELECTRIC-FIELD DOMAINS

In this section, we discuss briefly the model used to simulate the \(J-F_{\text{av}}\) characteristics of an entire TQL, including the domain formation on the basis of the \(J-F_{\text{per}}\) characteristics calculated for a single period. These equations were originally introduced for describing the transport properties of weakly coupled superlattices, in particular, the formation of EFDs related to a negative differential conductivity (NDC).\(^{29-31}\) In Ref. 32, we have applied these equations to describe the stationary \(J-F_{\text{av}}\) characteristics of THz QCLs and extended this model in order to predict also the optical output power of THz QCLs in the presence of EFDs. The relevant equations are given by (in analogy to Refs. 29–31)

\[
F_i - F_{i+1} = \frac{e}{\epsilon_0} (n_i - N_D), \quad (2)
\]

\[
e^d n_i \frac{d \theta}{d \phi} = J_{i-1} - J_{i-1}^{\text{leak}}, \quad (3)
\]

\[
\sum_{i=0}^{N_{\text{per}}} F_i = F_{\text{av}} N_{\text{per}}, \quad (4)
\]

where \(F_i\) denotes the driving field strength for the \(i\)-th period. The current density from the \(i\)-th to the \(i+1\)-th period is given by

\[
J_{i-1} - J_{i-1}^{\text{leak}} = \nu(F_i) \left[ n_i - n_{i+1} \exp \left( -\frac{e|F_i|}{k_B T} \right) \right], \quad (5)
\]

where \(\nu(F_i)\) denotes the field strength-dependent drift velocity of the free carriers in a single period \(i\). We have determined the \(\nu(F_i)\) values from our calculated \(J-F_{\text{per}}\) characteristics of a single period (cf. Sec. II) and assuming \(\nu(F) = J(F)/d_{\text{per}}(\epsilon N_D)\). For the contacts, we assume Ohmic boundary conditions, i.e., \(J_{0,1} = \sigma_0 F_0\) and \(J_{N,N+1} = \sigma F_N\), where \(\sigma = J_{\text{max}}/F_{\text{max}} = 0.047\, (\Omega \text{ cm})^{-1}\) denotes the contact conductance, \(J_{\text{max}}\) the current density, and \(F_{\text{max}}\) the field strength of the NDC-dominated maximum in the drift velocity.

VI. DISCUSSION OF THE CURRENT-VOLTAGE CHARACTERISTICS

A. Dark case

We analyze first the dark \(J-F_{\text{av}}\) characteristics of an entire TQL in the absence of any pumping (between the pump pulses), since the TQLs are optically pumped in a pulsed mode with a very low duty cycle, although electrically the TQL is permanently driven by a constant voltage. According to Eq. (5), the \(J-F_{\text{av}}\) characteristic is determined by the \(J-F_{\text{per}}\) characteristic of a single period. The calculated low-temperature \(J-F_{\text{per}}\) characteristic of a single period without MIR pumping is depicted in Fig. 4 by the circles. It shows a sharp pronounced tunneling resonance at \(F_{\text{per}} \approx F_{d} \approx -13\, \text{kV/cm}\), where the ground state of the MIR pump transition is in resonance with the final state of the MIR recovery process. The corresponding band structure diagram is shown in Fig. 5. For higher field strengths, \(|F_{\text{per}}| > |F_{d}|\), the calculated current densities (cf. circles in Fig. 4) are much lower than the values observed for the entire TQL (cf. the solid line with structured plateaus in Fig. 4). Therefore, we assume, in addition to the calculated intersubband tunneling current, also a leakage current through each period, which increases with increasing field strength (cf. dotted line in Fig. 4). We have already discussed such leakage currents for THz QCLs in Ref. 25. Furthermore, we assume that the calculated tunneling current resonance at \(-13\, \text{kV/cm}\) is much broader in real systems. We describe the total current density \(J_{\text{per}} + J_{\text{leak}}\) versus \(F_{\text{per}}\) (cf. dashed line in Fig. 4) through a single period by an Esaki-Tsu ansatz.\(^{29}\) Using these data in Eqs. (2)–(5), we calculate the \(J-F_{\text{av}}\) characteristic of the voltage-biased, unilluminated TQL, depicted in Fig. 4 by the solid line with flat plateaus. The up and down sweeps of this calculated \(J-F_{\text{av}}\) characteristic exhibit for medium field strengths between \(F_{\text{av}} \approx -12.7\) and \(-27\, \text{kV/cm}\) plateau-like structures, in agreement with the measured \(J-F_{\text{av}}\) characteristic (cf. the solid line with...
structured plateaus in Fig. 4). Within this region of monotonically increasing field strengths, 143 current jumps are calculated for an up sweep, which is close to the total number of periods and agrees well with the observed 136 current jumps for this TQL. The formation of EFDs in the entire TQL is related to the presence of NDC within each single period, caused by the tunneling resonance between the MIR pump ground state and the final MIR recovery state across the THz stage.

For the unilluminated TQL, we calculate additional tunneling resonances at lower field strengths. At $F_{\text{av}} \approx -8 \, \text{kV/cm}$, the final recovery state is in resonance with the upper THz state of the next period, and, at $F_{\text{av}} \approx -8.4 \, \text{kV/cm}$, there is a resonance between the pump ground state and the lower THz state of the next period. However, because these tunneling resonances include subband states between different periods, these calculated current resonances are smeared out in real systems, due to inelastic, incoherent scattering processes. Therefore, they are not included in the total current density through each period (cf. dashed line in Fig. 4) and do not affect the calculated $J-F_{\text{av}}$ characteristic of the entire TQL.

B. Cw MIR pumping

The question of whether MIR pumping can prevent the formation of tunneling-related EFDs is of central interest. In principle, any NDC can be reduced or prevented, if the corresponding transition is shunted. This stabilizes the potential drop across this transition, increases the minimum current within the bistable current region, and, therefore, reduces the NDC behavior. In the case of a TQL, it may be assumed that this shunting can be realized by intense $p$-polarized MIR pumping from the injector into the high-energy THz state, which is necessary in order to generate gain. However, an experimental verification of this shunt hypothesis cannot easily be performed under realistic experimental conditions, since a cw pumping can be applied only at small pump powers in order to prevent a strong heating of the TQL. At the same time, we have observed that a 1 W cw CO$_2$ pumping does not prevent the formation of EFDs. Therefore, we have applied pulsed pumping with large pulse powers and very low duty cycles (<0.1%) instead of cw pumping as described in Subsection VI C. Although, for very low duty cycles, the influence of the $p$-polarized pulsed MIR pumping on the time-integrated $J-F_{\text{per}}$ characteristics is expected to be very small, we have observed a pump-related switching of the metastable up sweep from the high-current to the low-current branch (cf. circles in Fig. 3).

For an analysis of cw pumping, we first calculate the drift velocity $v(F_{\text{per}})$ of the electrons for $p$-polarized light with 10.6 $\mu$m, as discussed in Sec. II. We assume that the leakage current (cf. dotted line in Fig. 4) remains unchanged for all pump powers. Figure 6 shows the $J-F_{\text{per}}$ characteristics for a single period calculated for pump power densities of 0, 5, and 10 MW/cm$^3$. These $J-F_{\text{per}}$ characteristics exhibit additionally a pronounced broad current maximum starting below $|F_{\text{d}}| = 10 \, \text{kV/cm}$ for very low pump powers (not shown), which shifts to $F_{\text{d}} \approx -11.3 \, \text{kV/cm}$ for pumping with 10 MW/cm$^3$. The sharp tunneling resonance at $F_{\text{d}} \approx -13 \, \text{kV/cm}$ of the dark $J-F_{\text{per}}$ characteristic remains nearly unchanged. Additional pump-related current maxima appear at higher field strengths $|F_{\text{per}}|$.

Figure 7 displays the $J-F_{\text{per}}$ characteristic of a single period calculated for an experimentally realistic cw pump power density of 0.25 MW/cm$^3$ at 10.5 $\mu$m. This characteristic exhibits clearly both the MIR pump-induced and the tunneling-induced current maxima at $F_{\text{per}} \approx F_{\text{d}} \approx -9.3 \, \text{kV/cm}$ and $F_{\text{per}} \approx F_{\text{d}} \approx -13 \, \text{kV/cm}$, respectively. Figure 8 indicates that the pump-induced current maximum is caused by a resonance between the 10.6 $\mu$m CO$_2$ pump beam and the principally dipole-forbidden intersubband transition $\text{MIR}_{\text{pump,forb}}$ from the injector ground state (cf. lowest thick line on the left side in Fig. 8) into the lower THz state (cf. second highest line in Fig. 8). At the resonance field strength $F_{\text{per}} \approx F_{\text{d}}$, the dipole moment of this dipole-forbidden transition is about one order of magnitude smaller than that of the dipole-allowed MIR pump transitions into the upper THz laser state (cf. Sec. II). However, with decreasing field strength $F_{\text{per}} < |F_{\text{d}}|$, the dipole moments of both the forbidden and the allowed MIR pump transitions approach each...
other. At $F_{\text{per}} \approx -9.3$ kV/cm, the dipole moment of the originally forbidden MIR pump transition increases to about 63% of that of the allowed transition ($D_{\text{MIR\,pump,\,forb}} = 0.094$ and $D_{\text{MIR\,pump,\,low}} = 0.15$). Therefore, for field strengths below the resonance value, the dipole-forbidden pump transition becomes rather dipole allowed and causes the current resonance at about $-9.3$ kV/cm. However, pumping via this transition does not generate either gain due to population inversion or GWI. Actually, cw pumping via this rather dipole-allowed intersubband transition leads to the formation of new EFDs within the $TQL$, indicated by the different sawtooth-shaped, plateau-like regions in the calculated $J-F_{\text{av}}$ characteristic of the entire $TQL$ (cf. solid lines in Fig. 7). This current maximum due to resonant MIR pumping is a new mechanism, which leads to a NDC different from that of sequential tunneling$^{29–31}$ or optically induced mechanisms.$^{33,34}$ From our calculations, we conclude that cw pumping (which cannot be realized experimentally due to the heating mentioned above) is not expected to prevent the formation of EFDs. Therefore, the homogeneous internal resonance field strengths and, consequently, THz gain cannot be reached by resonant pumping, in contrast to the shunt hypothesis mentioned above.

C. MIR pulse pumping

For the simulation of the time-integrated $J-F_{\text{av}}$ characteristics of pulsed, MIR-pumped $TQLs$ (cf. lines in Fig. 3), we assume in our numeric simulation (i) a long start interval, where the $TQL$ is in the dark for 300 ns, (ii) followed by a p-polarized MIR pump pulse with 10.6 µm, a power density of 0.25 MW/cm$^3$, and different pulse lengths of $\tau_{\text{pulse}} = 0$, 0.25, 2.6, 4, 5, 300, and 730 ns, and (iii) finalized by a long dark interval (300 ns). The different operation conditions (dark or pumped) of the $TQL$ were taken into account by including the corresponding calculated $J-F_{\text{av}}$ characteristics (dark or MIR pumped) of a single period. For the simulation of cw pumping, we select a step width of 0.5 ns; for the 50 ns CO$_2$ laser pulse, 0.1 ns; and for the 3 ps FELBE pulse, $1 \times 10^{-3}$ ps. The calculated $J-F_{\text{av}}$ characteristics are plotted in Fig. 9. They show that, for MIR pumping with short pulses $\tau_{\text{pulse}} \leq 4$ ns, the up sweep (cf. three solid lines on the bottom for 0, 2.6, and 4 ns in Fig. 9) of the $J-F_{\text{av}}$ characteristics remain almost close to the high-current branch of an up sweep of the dark $J-F_{\text{av}}$ characteristic (cf. lowest solid line in Fig. 9). As predicted by this calculation, we observed that MIR pumping with the very short pulses of the free-electron laser FELBE does not modify the dark, time-integrated $J-F_{\text{av}}$ characteristics. However, for pump pulse lengths between 4 and 5 ns, the calculations predict an abrupt switching of the up sweep from the high-current branch to a low-current branch. Above 5 ns up to about 1 μs (cf. top three solid lines for 5, 300, and 730 ns in Fig. 9), this switching behavior remains relatively stable. Independent of the length of the pump pulse, all down sweeps remain on the low-current branch of the dark $J-F_{\text{av}}$ characteristics (black circles). This calculated switching behavior is qualitatively in good agreement with the
experimental findings for pulsed pumping with the $p$-polarized CO$_2$ laser (cf. lines in Fig. 3).

In contrast to our simulations for cw pumping, which induced new EFDs, the temporal interplay between EFDs related to sequential tunneling (dark case, no pumping) and domains induced by MIR pumping (during the pump interval) leads, in pulse-pumped TQLs, to the observed switching behavior for an up sweep from the low-current branch in the $J$-$F_{av}$ characteristics (cf. Figs. 3 and 9). This switching behavior is accompanied by a change of the field strength values of the dominating high- and low-field domains from the dark case ($-27$ and $-11$ kV/cm, respectively) to the pulse-pumped case ($-24$ and $-12$ kV/cm, respectively), independent of $F_{av}$. At the same time, the depletion layer between the high- and low-field domain shifts as a function of $F_{av}$. This layer is localized in the dark case between the 89th and 90th period and switches between the 113th/114th period for an up sweep at $F_{av} = 21$ kV/cm. Finally, note that the shape of the switched $J$-$F_{av}$ characteristics (cf. Figs. 3 and 9) is mainly determined by the shape of the $\nu(F)$ relation used in Eq. (5), which is influenced by the field dependence of the transition energy $E_j(F_{pes})$ (Stark shift), the field-dependent dipole moment $D_j(E_j, F_{pes})$, the averaged electron relaxation time $\tau(E_j)$, and the photon density $S$ [Eq. (1)]. In addition, the doping density $N_D$, the number of periods, the length of each period, and the contact resistance [Eqs. (2)–(5)] also affect the shape of the switched $J$-$F_{av}$ characteristics. The time constant of the TQL $[\tau_{TQL} = \nu_{max}/(\epsilon F_{max}) \approx 2.4$ ns] has no direct effect on the shape of the switching behavior.

**D. Further pump-related effects**

Further effects of MIR pumping on the $J$-$F_{av}$ characteristics, which are not included in our model, may occur:

(i) The time constant for the pump-induced charge transfer between different EFD regions can be larger than the length of the pump pulse caused, e.g., by long-living GaAs/(Al,Ga)As interfaces states. However, time-resolved photocurrent measurements (not shown) indicate that these interface states are not relevant for the investigated TQLs.

(ii) The 10.6 $\mu$m pump beam may be affected by interferences within the waveguide of the TQL (the optical width of the waveguide is about 3 times the wavelength of the pump beam), and, therefore, additional inhomogeneities occur in the direction of the applied field. Because our calculated current densities agree rather well with the measured values, we believe that interference fringes in the investigated TQLs are averaged out.

(iii) Finally, we expect also for purely $p$-polarized pumping an $s$-polarized component due to the pumping through 45° facets. However, $s$-polarized MIR pumping causes a strong switching from the high- into the low-field domain for an up sweep (cf. triangles in Fig. 3). This effect cannot be explained within our wavevector-independent model, since in-plane excitations are not included (cf. Sec. II). Strong $s$-polarized pumping causes in-plane excitations accompanied by electron scattering on impurities, interfaces, etc. The following relaxation of electrons from these states with finite wavevector can be realized by intersubband transitions.

**VII. SUMMARY AND CONCLUSION**

We have discussed a design for an MIR-pumped, electrically driven TQL, which consists of cascaded periods, where each period is formed by a THz stage coupled to an injector. Each single THz stage of this TQL is expected to exhibit THz gain, which is based on a population inversion as well as on quantum coherence effects for MIR pumping at the resonance field strength according to numerical simulations and the model introduced by Waldmueller et al.\textsuperscript{22} However, we observe the formation of EFDs in the current-voltage characteristics for both the dark as well as the MIR pulse-pumped TQLs already at a field strength below the designed operating field strength, which prevents the realization of the conditions for THz lasing.

We explain the formation of EFDs to be related to the presence of NDC, which are in the dark case caused by sequential tunneling through the THz stages, which act as barriers with a low tunneling probability. In the illuminated case, an additional NDC is induced by MIR pumping between the pump ground state and the lower THz state. This pump intersubband transition is dipole forbidden at the resonance field strength, but becomes, to some extent, dipole allowed at lower field strengths. Both the tunneling-related as well as the MIR pump-related EFDs are a consequence of the field strength-dependent coupling of the THz stage to an injector in an entire TQL, which is not treated in Waldmueller’s model\textsuperscript{22} for a single THz stage. The switching behavior of the current-voltage characteristic for pulsed pumping is explained by the temporal interplay between the tunneling-induced and pump-induced EFDs.

The challenge for future designs of TQLs is to avoid resonances between subband states across the THz stage at field strengths below its resonance value $|F| < |F_0|$. Furthermore, the dipole moment of the allowed MIR pump transition has to be increased at $F = F_0$ in order to increase the pump efficiency, and, at the same time, the dipole moment of the forbidden pump transition at $|F| < |F_0|$ has to be decreased in order to prevent a resonance between the MIR pump beam and an undesired intersubband transition.

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\textsuperscript{1}B. S. Williams, *Nature Photon.* 1, 517 (2007).


