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ABSTRACT
A recent experiment [Hortelano et al., Semicond. Sci. Technol. 32, 125005 (2017)] reported a rectification effect that appeared in curved narrow ballistic channels of a two-dimensional electron gas when the strength of a magnetic field applied to the channels was tuned. The phenomenon was reproduced by classical billiard simulations as resulting from a transmission asymmetry caused by diffuse boundary scattering. However, this manifests breakdown of a commonly used simple model for diffuse boundary since the magnetic-field dependence of the transmission in two-terminal geometries has to be symmetric in equilibrium. We demonstrate here that this tendency of the system predicted by the billiard simulations is a real transmission asymmetry effect that emerges in the nonequilibrium transport. We perform nonequilibrium quantum-mechanical simulations with taking into account Coulomb repulsion. Experimental observations are presented to demonstrate consistencies with the numerical results.

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I. INTRODUCTION
We have recently investigated the DC voltage generated by an rf excitation in circular ballistic bends created using the two-dimensional electron gas (2DEG) in a GaAs-(Al,Ga)As heterostructure. A finite voltage that increased its amplitude with increasing the excitation intensity emerged in the presence of a magnetic field. The voltage was attributed to a rectification effect in the circular bends induced by the diffuse component in the scattering of ballistic electrons from the channel boundary. The mechanism is summarized as follows. When the cyclotron radius $r_c$ of the 2DEG coincides with the curvature radius $R$ of the channel, the collision of the electrons with the channel boundary becomes scarce; see the red trajectory in Fig. 1(a). The boundary reflection is, on the other hand, frequent when the cyclotron orbit turns in the direction opposite to the curving of the channel for the reversed magnetic-field direction, or equivalently for electrons moving in the opposite direction of the channel, as shown by the blue trajectory in Fig. 1(a). A transmission asymmetry arises between the clockwise and counterclockwise propagations in the circular channel, when the diffuse boundary gives rise to backscattering. Under the $rf$ excitation of the channel, the voltage associated with the symmetric transmission cancels to be zero on average. The rectification due to the transmission asymmetry, in contrast, generates a DC voltage. Remarkably, the voltage in the experiment remained to be finite even in the absence of the $rf$ excitation.1 The rectification effect is indicated to be so sensitive that the electromagnetic noises that generally exit in the environment were rectified. The device can thus be utilized to harvest energy by serving as a voltage source that extracts a DC voltage out of the electromagnetic fluctuations. In addition to the natural electrical noises in environment, the electrical equipment in modern society emits electromagnetic fluctuations. Such an energy loss can be partially recovered using the rectification device.

Numerical simulations using a classical billiard model were carried out in Ref. 1 to justify the aforementioned phenomenological explanation. Although the system was thereby demonstrated to have a strong tendency to exhibit the transmission asymmetry, the numerical results are, strictly speaking, incorrect. The billiard simulation predicts that the transmission probabilities in the circular bend at magnetic fields $ \pm B$ which satisfy the condition $r_c = h k_F/e |B| = R$ are not identical when non specular boundary scattering is incorporated by...
reflecting the electrons incident to the channel boundary in random directions. Here, $k_F = (2\pi n_s)^{1/2}$ is the Fermi wavenumber with $n_s$ being the sheet concentration of the 2DEG. However, the transmission probability is required to be symmetric in two-terminal geometries under magnetic-field reversal. In two-terminal geometries, the transmission probability $T$ and the conductance $G$ of the system are equivalent as they are related to each other by the Landauer formula $G = (2e^2/h)T$. As highlighted by the discussions triggered by the unintended experimental observation in an Aharonov-Bohm loop by Yacoby et al., the magnetic-field dependence of the two-terminal conductance has been established to be symmetric. The symmetry with respect to magnetic-field reversal $G(B) = G(-B)$ is satisfied also by the transmission probability as $T(B) = T(-B)$. We have, therefore, encountered a situation where the commonly used simple model for diffuse boundary scattering is not reliable.

In the first part of this paper, we show that an asymmetry emerges in the magnetic-field dependence of the transmission when fully quantum-mechanical simulations are extended to the nonequilibrium regime. The nature of the system to exhibit the asymmetry predicted by the billiard simulation develops to a real effect when the restriction of the symmetric magnetic-field dependence is lifted for the nonequilibrium transport in the presence of electron-electron interactions. We then compare further predictions of the quantum-mechanical simulations with experimental results in the rest of the paper. Some additional experimental observations are also presented.

II. NUMERICAL SIMULATIONS

In this section, we numerically investigate the ballistic electron transport in curved channels. We clarify the conditions that are necessary to realize the transmission asymmetry in the course of the discussion.

Billiard simulations treating the 2DEG as classical particles have proved to be a powerful method for explaining the ballistic transport phenomena in mesoscopic structures. To take into account the diffuse component of the boundary scattering, a fraction $1 - p$ of the electrons incident to the channel boundary was assumed to be scattered in random directions in the simulations in Ref. 1. Here, $p$ is the specularity in the boundary reflection. The diffuse boundary was assumed in the curved part of the channel, which is shown by the green curves in Fig. 1(a). With this treatment of the diffuse boundary scattering, the transmission became asymmetric under magnetic-field reversal when $p < 1$. In the previous reports using this model, the symmetric magnetic-field dependence was guaranteed by the symmetric geometry of the channels. The breakdown of the model for the diffuse boundary in the fashion of the magnetic-field-reversal asymmetry presumably occurs generally for nonsymmetric channels. A direct consequence of the way the diffuse boundary scattering is treated in the model is that the momentum parallel to the channel boundary for the scattered electrons is on average zero. The momentum conservation is hence violated even on average when the angular distribution of the incident electrons is not symmetric, which will be almost certainly the case for ballistic transport except when the transport is chaotic. The magnetic-field-reversal asymmetry plausibly occurs when the degree of the violation differs for the two magnetic-field directions for asymmetrical shaped channels.

The breakdown of the magnetic-field-reversal symmetry due to the random boundary reflection can be understood in terms of time-reversing operation. With respect to the transmission probabilities in multiterminal geometries, the time-reversing operation leads to a relationship that reversing the propagation direction is equivalent to reversing the direction of an external magnetic field

$$T_{ij}(B) = T_{ji}(-B),$$

where $T_{ij}(B)$ is the transmission probability from lead $i$ to lead $j$ in the presence of a magnetic field $B$. In two-terminal geometries, $T_{ij}$ and $T_{ji}$ are the same, and so one obtains the symmetric magnetic-field dependence $T(B) = T(-B)$. The reflection to random directions at the channel boundary in modeling the diffuse boundary breaks the time-reversing operation for the ballistic trajectories. The symmetric magnetic-field dependence is no longer required in such a case. If the diffuse boundary is taken into account as, for instance, a local meandering of the channel boundary, the ballistic trajectories are unchanged under simultaneous reversal of momentum and magnetic field as the boundary reflection is microscopically specular. The transmission probability obtained by such a modeling of the diffuse boundary is symmetric under magnetic-field reversal.

A. Equilibrium quantum transport

We perform in this subsection fully quantum-mechanical simulations that correspond to the above classical situation. The equilibrium transmission in two-terminal geometries will be confirmed to be symmetric under magnetic-field reversal even when diffuse boundary scattering is present. We have calculated the transmission probability using the lattice-Green’s-function method. (The theoretical details are presented in the Appendix.) The circular channels are approximated using a square tight-binding lattice having the lattice parameter $a$, as illustrated in Fig. 1(b). We introduce nonpeculiarity in the boundary reflection by altering the onsite potential.

**FIG. 1.** Semicircular ballistic bend. The electron trajectories relevant for the transmission asymmetry when the cyclotron radius coincides with the curvature radius $R$ of the channel are shown in (a). An electron is injected from the left- and right-hand side leads for the red and blue curves, respectively. The width of the channel is $W$ and a magnetic field $B$ is applied perpendicular to the channel. The boundary reflection is assumed to be partly specular at the green sections. The tight-binding model to simulate the quantum transmission in the situation in (a) is illustrated in (b). The lattice constant of the square lattice is $a$. The number of transverse lattice sites in the leads is $N$, Random potentials are assumed at the boundary lattice sites indicated by green.
by $\delta V$ on the boundary lattice sites, which are shown by the green dots in Fig. 1(b). The disorder $\delta V$ is randomly distributed within an interval $[-D/2, D/2]$.

In order to decide the disorder strength to be used in the following simulations, the variation of $T$ with $D/t$ was calculated, as shown in Fig. 2. Here, $t = \hbar^2/(2ma^2)$ is the nearest-neighbor hopping amplitude in the tight-binding model. The above-mentioned boundary disorder was assumed for the filled circles. The case of bulk disorder was also considered for comparison as shown by the open circles, where the random potential was imposed for all the lattice sites. In these simulations, the number of occupied modes in the leads attached to the curved channel was 12 and 145301 (2019); doi: 10.1063/1.5064689

The width of the leads is given as $W = (N_l + 1)a$, with $N_l (= 49)$ being the number of transverse lattice sites in the leads. The transmission decreases monotonically for the bulk disorder when the randomness increases, leading to the localization regime. Although the transmission initially decreases similarly for the boundary disorder, it eventually increases as the disorder further increases. We attribute this behavior to quantum-mechanical expulsion of the wavefunction from the disordered channel boundary. When large random potentials are imposed at the boundary, the repulsion for the electron wavefunction by negative potentials is more effective than the attraction by positive potentials. This is the consequence that an electron cannot be localized within a narrow potential segment represented by a single lattice site due to the quantum confinement effect. The wavefunction is pushed away from the boundary, and so the electron does not experience the full impact of the boundary disorder.

Strong localization was demonstrated numerically to occur when the diffuse boundary was incorporated as random fluctuations in the width of a narrow channel. The boundary scattering does not lead to the localization in Fig. 2 as the length of the channel is much shorter than for the localization effect to be relevant. We point out that the quantum confinement effect is not significant in Fig. 2 as a large number of quasi-one-dimensional subbands are occupied below the Fermi level in the channel. Our simulations thus correspond to the classical situation investigated in Ref. 1.

The magnetic-field dependence of the transmission is shown in Fig. 3 for a number of cases of the parameters. The filled and open circles correspond to channels with the boundary and bulk disorders, respectively. The disorder strength was set to the values indicated by the dotted line in Fig. 2. That is, the electron transport is maintained to be nearly ballistic. As a consequence of the two-terminal geometry, the transmission is exactly symmetric under magnetic-field reversal. The transmission probability was calculated by averaging over a large number ($5000–10,000$) of disorder realizations. The quantum fluctuations associated with the random potentials were averaged to be smaller than the size of the symbols in Fig. 3.

The geometrical matching $r_c = R$ in the curved channel does not appear to provide a feature in the transmission characteristics. Although a large dip indicated by the arrow is present, the feature does not survive when the channel parameters are altered. That is, some geometrical effects may appear incidentally even in the disorder-averaged transmission characteristics. However, no systematic behavior originating from the curved channel geometry seems to play a role in the equilibrium transmission. It should be pointed out in this regard that diffuse boundary scattering in

![FIG. 2. Dependence of equilibrium quantum transmission on disorder strength $D$. Boundary and bulk disorders are assumed for the red and green circles, respectively. The transmission probabilities are averaged over 500 random potential realizations. The parameters are $k_F W = 12.2$, $R_t = 7$, and $N_l = 49$. The disorder strengths are set to the values indicated by the dotted line for the simulations performed in Figs. 3 and 4.](image)

![FIG. 3. Magnetic-field dependence of equilibrium quantum transmission in semicircular channels. The abscissa $R/r_c$ with $R$ and $r_c$ being, respectively, the curvature radius of the channel and the cyclotron radius is proportional to the magnetic field. The transmission is exactly symmetric with respect to magnetic-field reversal. The parameters $k_F W / \pi$ and $R/W$ were varied as summarized at the bottom of the panel with $N_l = 49$. The filled and open circles correspond to, respectively, the boundary and bulk disorders with the disorder strengths $D/t = 1.2$ and 0.16. The uncertainty in the transmission amplitude due to universal conductance fluctuations (UCFs) associated with the random potential realizations is smaller than the size of the symbols. The dotted lines mark the magnetic fields corresponding to the condition $W/r_c = 0.55$ achieved for the red and green curves.](image)
narrow long wires is known to cause a resistance peak at a condition \( W/r_c = 0.55 \).\(^{29}\) The condition is satisfied for our numerical results at the magnetic field indicated by the dotted bars. We find even no feature corresponding to this resistance peak. The channels in our simulations are suggested to be not sufficiently long to produce this scattering effect.

The restriction of the symmetric magnetic-field dependence is no longer relevant in multiterminal geometries as the symmetry of the transmission is given by Eq. (1). We have, therefore, examined a system where two leads, whose width is \( W/2 \), are attached to each end of the curved channel as illustrated in the inset of Fig. 4(f) to see if such a modification to a four-terminal geometry affects the symmetry of the transmission. We emphasize that application of the Büttiker formula\(^{21}\) can handle only a small lattice system, we use a model system shown in the manner of the \( G = eI/(\mu_j - \mu_i) \) does not give the Landauer formula. Here, \( \mu_i, \mu_j \) and \( \mu_i \) are the current at lead \( i \), the number of occupied modes in lead \( i \), and the chemical potential of the reservoir attached to lead \( i \), respectively. It is, therefore, not obvious whether the transmission asymmetry due to diffuse boundary scattering remains to be absent under the alteration to a four-terminal geometry. It should be reminded that a continuous Aharonov-Bohm phase shift was realized experimentally when the two-terminal geometry by Yacoby et al.\(^{17}\) was modified to measure a four-terminal resistance.\(^{29}\)

As manifested in Figs. 4(a)–4(d), the variation of the transmission probabilities with magnetic field is no longer symmetric. Although the probabilities were averaged over \( 10^4 \) disorder realizations, \( T_{ij}(B) \) exhibited fluctuations. The fluctuations are presumably due to the quantum interference associated with the quantum-mechanical scattering of an electron at the branched channel ends. As no qualitative difference is noticed between the boundary and bulk disorder cases concerning the fluctuations, we conclude that the sought-after diffuse boundary effect at \( R/r_c = 1 \) is absent.

Four-terminal resistances \( R_{14,23} \) and \( R_{13,24} \) are calculated in Fig. 4(e). Here, \( R_{nm} \) means that leads \( m \) and \( n \) are used for the current and the voltage difference is measured between leads \( k \) and \( l \). Note that \( R_{13,24} \) contains the Hall effect, giving rise to the antisymmetric background. The four-terminal resistances exhibit asymmetric magneto-resistance fluctuations. Nevertheless, the fluctuations are attributed to the above-mentioned resonances due to multiple scattering between the ends of the curved channel. The absence of the transmission asymmetry resulting from the diffuse boundary even when the transmission probabilities are not symmetric functions of magnetic field is, in fact, reasonable considering the following observation. Although the individual transmission probabilities are not symmetric under magnetic-field reversal, the total transmission probability through the channel \( T_{13} + T_{14} + T_{23} + T_{24} \), which is shown in Fig. 4(f), remains to be exactly symmetric. The geometrical modification, therefore, does not alter the fundamental symmetry properties of the transmission in an essentially two-terminal channel.

### B. Nonequilibrium quantum transport

Diffuse boundary scattering is unable to produce a transmission asymmetry in the equilibrium transport, as we described in Sec. II A. We thus extend the calculation to the nonequilibrium transport regime.\(^{20–22}\) We employ the nonequilibrium-Green’s-function technique developed by Nonoyama and Oguri.\(^{23}\) As we show below, the calculations of the nonequilibrium quantum transport that produce the transmission asymmetry are numerically time-consuming. We cannot satisfactorily fulfill the conditions \( W \gg \lambda_F \gg a \) that are appropriate for our situation. Here, \( \lambda_F \) is the Fermi wavelength. Since we can handle only a small lattice system, we use a model system shown in Fig. 5(a) throughout the simulations in this subsection instead of the realistic lattice system employed in Sec. II A. [Note that Fig. 5(a) illustrates the exact lattice system to be used in the following simulations.] To be specific, we cannot avoid the influences of the quantum confinement as \( W \gg \lambda_F \) is not sufficiently satisfied. We thus assume the rectangular geometry for simplicity rather than the circular geometry that is ideal for the rectification effect. On the other hand, our numerical results will demonstrate that the rectification effect is
The magnetic-field-reversal symmetry is finally broken when a self-consistency of the potential is imposed in the nonequilibrium perspective, as previously discussed in Refs. 24 and 25. The reason for the asymmetry can be easily understood in the situation illustrated in Fig. 5(b). We assume that a strong magnetic field is applied to the channel, and so the transport occurs via the edge states propagating along the upper and lower boundaries of the channel. The channel is connected to reservoirs at the two ends. The chemical potentials of the reservoirs are different \( \mu_L \neq \mu_R \) as we consider the nonequilibrium transport. The difference is related to the bias voltage \( \mu_L - \mu_R = eV \). At one direction of the magnetic field corresponding to the case illustrated in Fig. 5(b), the edge state at the upper (lower) boundary is occupied up to \( \mu_L (\mu_R) \). The edge state is, however, occupied up to \( \mu_L (\mu_R) \) when the magnetic field is reversed. The charge density at the boundary thus changes with the magnetic-field reversal. The nonequilibrium transport in

- FIG. 5. (a) Tight-binding model used for nonequilibrium simulations. While the illustration corresponds exactly to the lattice system used for the simulations in Fig. 6, not all the lattice sites are shown for the sake of clarity. The bias voltage \( V \) is assumed to drop linearly along the blue lines. Disorder potentials are imposed for the boundary lattice sites shown as the green dots. The orange curve shows the cyclotron orbit at the magnetic field for the asymmetry peak in Fig. 6. (b) Nonequilibrium edge state transport at high magnetic fields to highlight the role of the space charge potential on magneto-transmission asymmetry. The difference of the chemical potentials is related to the bias voltage \( V \) applied to the channel as \( \mu_L - \mu_R = eV \). The edge states at the upper and lower boundaries are occupied up to the chemical potentials \( \mu_L \) and \( \mu_R \), respectively.

- FIG. 6. Magnetic-field dependence of nonequilibrium quantum transmission. The parameter \( \beta \) is related to the magnetic field \( B \) as \( \beta = Ba^*/(\hbar/e) \). The nonequilibrium conductance \( G \equiv I/V \) normalized by the conductance unit \( 2e^2/h \) is shown by the blue curve. The equilibrium transmission probability, i.e., for \( V = 0 \), is also shown by the green curve for comparison. The nonequilibrium current is obtained by accumulating the contributions over the energy window corresponding to the bias voltage. The amplitude of the quantum fluctuations hence decreases with increasing the bias voltage due to the averaging over the energy similar to the effect of Fermi distribution smearing at finite temperatures. The nonequilibrium current in this circumstance is symmetric under magnetic-field reversal since the symmetric characteristic similar to the equilibrium transmission at the individual energies is retained.

The numerical results are plotted in Fig. 6. Here, the parameter \( \beta \) describes the magnetic field as \( \beta = B a^*/(\hbar/e) \). The nonequilibrium transport within the scheme described above is still symmetric when the magnetic field is reversed. In Fig. 6(a), the nonequilibrium conductance \( G \equiv I/V \) normalized by the conductance unit \( 2e^2/h \) is shown by the blue curve. The equilibrium transmission probability, i.e., for \( V = 0 \), is also shown by the green curve for comparison. The nonequilibrium current is obtained by accumulating the contributions over the energy window corresponding to the bias voltage. The amplitude of the quantum fluctuations hence decreases with increasing the bias voltage due to the averaging over the energy similar to the effect of Fermi distribution smearing at finite temperatures. The nonequilibrium current in this circumstance is symmetric under magnetic-field reversal since the symmetric characteristic similar to the equilibrium transmission at the individual energies is retained.
The asymmetry of the transmission is caused by the quantum-mechanical reflection from the bending of the channel. We cannot, however, rule out the possibility that this peak is merely a part of the universal conductance fluctuations (UCFs). The quantum fluctuations were averaged for the case shown by the red circles. The sign changes in the orange curve in Fig. 6(b) disappeared in the ensemble-averaged characteristics as UCF were suppressed. This provides us confidence that the asymmetry peak indicated by the arrow originates from the curved channel geometry.

The nonequilibrium quantum simulations that enable us correctly predicting the magneto-transport asymmetry are time-consuming. The recursive calculations of the lattice Green’s function to determine the nonequilibrium current have to be carried out at a number of energies within the bias window. The procedure is then repeated until the self-consistency of the potential is achieved. Furthermore, some lattice Green’s functions need to be computed additionally for all energies up to the quasi-Fermi level to obtain the mean-field charge density. The self-consistency of the potential becomes increasingly difficult to accomplish when the magnetic field is strong. For this reason, the number of the disorder realizations N used to create the plots in Fig. 6 decreased rapidly for high magnetic fields as plotted in Fig. 6(c). The consequent incomplete averaging of the quantum fluctuations is clearly recognizable for β > 0.02.

III. EXPERIMENTAL RESULTS AND DISCUSSION

We now examine experimental findings paying attention to the correspondence with the numerical results of the nonequilibrium quantum simulation. Experimental manifestations of the rectification effect originating from the magnetic-field-induced transmission asymmetry are presented in Fig. 7. The inset shows a typical device fabricated from a GaAs-(Al,Ga)As heterostructure using electron-beam lithography and Ar ion milling. We point out that the Ar ion milling makes the boundary reflection considerably nonspecular. The mobility and concentration of the 2DEG at a temperature of 0.3 K prior to the nanostructuring were 180 m²/V s and 2.3 × 10¹⁵ m⁻², respectively. The electron transport in the device is ballistic as the mean free path (14 μm) is much longer than the length of the curved channel. In GaAs-(Al,Ga)As heterostructures, the lateral sidewall depletion in mesa-etched channels is large. The depletion width was estimated to be about 0.3 μm in the case in Fig. 7. The conduction channel was thus considerably narrower than the geometrical width and, moreover, the actual shape of the curved channel was roughly semicircular as the depletion is more pronounced at rectangle corners of the mesa. The DC voltage was measured between the leads attached to the ends of the curved channel, while the magnetic field was applied perpendicular to the channel.

As one finds in Fig. 7, a finite voltage emerged when the magnetic field was increased. The magnetic-field dependence of this voltage is approximately antisymmetric despite the two-terminal geometry. It needs to be pointed out that a magnetic-field-independent background was present in addition to the magnetic-field-induced voltage. The background increased its magnitude as the sample temperature T was lowered. The magnetic-field-independent background is thus attributed to the thermovoltage generated along the measurement wires. The voltage induced by the magnetic field peaked

![Graphical representation](image-url)  
**FIG. 7.** Magnetic-field dependence of DC voltage induced in a two-terminal ballistic bend. The inset shows the scanning electron micrograph of a typical device fabricated in a GaAs-(Al,Ga)As heterostructure. The DC voltage V was measured between the leads attached to the curved channel when the magnetic field B was applied perpendicular to the channel. The measurements were carried out without an external rf excitation at temperatures T of 10 and 77 K for the blue and red curves, respectively.
at $B = 0.14 - 0.2 \, T$ (at $T = 10 \, K$) due to maximal development of the rectification effect. The cyclotron radius is $390 \, nm$ at $B = 0.2 \, T$, which is comparable with the anticipated curvature radius of the channel.

The measurements in Fig. 7 were carried out without applying an rf excitation. (The rectification effect under an rf excitation will be shown below.) The nonequilibrium excitation that led to the voltage with the antisymmetric magnetic-field dependence was unintentional. That is, the voltage was generated by rectifying the electromagnetic noises in the environment. The remarkable sensitivity of the rectification effect is, therefore, promising for extracting energy from environmental electromagnetic noises. The red curve in Fig. 7 was obtained when a device was dipped in liquid nitrogen. The rectification effect is unambiguously evidenced to take place at a temperature of $77 \, K$. The temperature dependence is rather weak at the low temperatures reflecting the fact that the transport phenomenon originates from classical ballistic transport and thus depends predominantly on the mean free path. Although the phonon scattering reduces the electron mobility rapidly at high temperatures, the operation temperature will be raised further by miniaturizing the device.

In applying the rectification effect for energy recovering, it is important to be able to increase the output amplitude from the devices. We demonstrate in Fig. 8 the addition laws for the rectification effect. Prior to that, we point out that the output voltage from individual devices plotted in the upper panel is much larger than that in Fig. 7 as the devices were exposed to an rf excitation. The antisymmetric magnetic-field dependence of the output voltage and the magnetic-field value for the voltage peak were nearly unchanged under the external excitation. Merely the voltage increased its magnitude when the excitation was strengthened. (Note that a systematic study of the dependence of the asymmetry amplitude on the rf excitation intensity was presented in our previous publication.) These characteristics support our interpretation that the voltage in Fig. 7 was produced by rectifying the electromagnetic noises that inevitably existed in the environmental background.

Concerning the measurement setup, we paid attention in applying the excitation to the devices. To avoid generating a voltage/current by the DC offset in the output of the rf generator, the excitation was not applied directly to the devices. Instead, a metal pad which was placed in the vicinity of the devices but was isolated from them with high resistances was contacted by the rf generator. The semicircular ballistic channels of the devices were excited indirectly via the capacitive coupling at an rf frequency of $50 \, MHz$. The actual excitation strength was estimated to be about 3 to 4 orders of magnitude smaller than the nominal value ($V_{rf} = 2 \, V$).

We compare in the lower panel of Fig. 8 the output voltages when two devices were connected with each other in various ways. In summarizing the results, the device is found to work as a voltage source. The output signal can thus be multiplied in amplitude by connecting a number of devices in series. As shown by the red and blue curves, the voltages from individual devices added together and subtracted from each other when the two devices were connected in series with symmetric and antisymmetric configurations, respectively. When the two devices were connected parallel to each other, the voltage was significantly reduced in amplitude, as shown by the magenta and green curves. We attribute this behavior to the partial short-circuiting of the open-circuit voltage of one device by the small channel resistance of the other device. The output was the smallest for the antisymmetric parallel configuration as the voltages from the two devices additionally canceled each other.

Figure 9 shows an example when up to three devices were connected in series. Here, the voltage swing between the antisymmetric peaks at the positive and negative magnetic fields was evaluated. The peak-to-peak amplitude in the voltage is compared when the number $N$ of devices connected in series was varied between 1 and 3. The voltage in individual devices ($N = 1$) is shown in the colors red, green, and blue. The measured voltages when $N = 2$ and 3 are seen to be nearly the same to the expectations determined by adding the individual voltages. The measurements in Fig. 9 were performed without an intentional rf excitation. The addition law is confirmed to be the same regardless of the nature of the excitation. This is not obvious as the frequency range of the unintentional background noises is totally unknown. In addition, because the signal in the absence of an intentional excitation is small, the output can be seriously affected by the impedances of the devices. The latter may explain the relatively large difference between the observed and expected values for $N = 2$ in Fig. 9.

One notices in the experimental magnetic-field dependencies in Figs. 7 and 8 that the rectified voltage did not decrease to zero but saturated at finite values when the magnetic field further increased beyond the value for the voltage peak. The classical
The oscillatory behavior of the transmission asymmetry predicted by the quantum simulation was observed experimentally, as shown in Fig. 10. In the upper panel, the two-terminal voltage of a curved channel is plotted together with the four-terminal resistance of the channel. (The measurement configurations can be found in Ref. 1.) To clarify the origin of the voltage in the quantum Hall regime, we have decomposed the voltage to the asymmetric and antisymmetric components as shown in the lower panel by the red and blue curves, respectively.

FIG. 10. Transmission characteristics in the quantum Hall regime. The two-terminal voltage and the four-terminal resistance of a narrow curved channel at a temperature of \( T = 4.2 \) K are plotted in the upper panel. The Landau level filling factors \( \nu \) associated with the minima in the Shubnikov-de Haas oscillation are indicated. The DC voltage was measured in the absence of an external rf excitation. The resistance was measured using the lock-in technique. In the lower panel, the voltage is decomposed to the symmetric and antisymmetric components in terms of the magnetic-field dependence, as shown by the red and blue curves, respectively.
and blue curves, respectively. The antisymmetric component is attributed to the nonequilibrium transmission asymmetry that we discussed above. The symmetric component plausibly originates from the voltage that can appear when the channel resistance is extremely large in the quantum Hall regime. Instead of screening the rf voltage by the large conductivity of the high-mobility 2DEG, the channel acts as an antenna and captures the rf voltage when the 2DEG exhibits the quantum Hall effect. In other words, the cancellation of the voltage induced by the rf excitation for the symmetric transmission was not perfect in the experimental situation. The curve of the symmetric component in the quantum Hall regime indeed resembles that of the resistance. The antisymmetric component in Fig. 10 changed the sign from negative to positive when the magnetic field increased between the quantum Hall states corresponding to the Landau level filling factors ν = 4 and 2, in agreement with the numerical result. On the other hand, the opposite transitions occurred at magnetic fields considerably lower than those for the vanishing resistance. This discrepancy is attributed to the fact that the edge-state transport was not sufficiently reflectionless in the experiment. The simultaneously existing voltage having the symmetric magnetic-field dependence may also affect the rectification characteristics in the form of a bias effect.

IV. CONCLUSIONS

We have investigated the antisymmetric magnetic-field dependence of the transmission properties in two-terminal curved ballistic channels. Quantum nonequilibrium simulations have produced the asymmetry when the usual restriction of the symmetric magnetic-field dependence in two-terminal geometries is lifted in the presence of electron-electron interactions. The asymmetric transport is manifested experimentally as a voltage rectification effect when the cyclotron orbit of the conduction electrons coincides with the curvature radius of the curved channels. We have shown that the rectified voltage increased in amplitude when multiple devices were connected in series. Together with the remarkable sensitivity of the rectification effect, the transport phenomenon has thus been demonstrated to be attractive for energy harvesting. A consistency between the experimental and numerical results has been observed in high magnetic fields where an oscillatory behavior of the transmission asymmetry was caused by the scattering of edge states during the magnetic depopulation of the quantum Hall states.

APPENDIX: SIMULATION METHODS

The transport channels are approximated using a square lattice having a lattice parameter of \(a\), see Fig. 1(b). We use the tight-binding Hamiltonian

\[
H_0 = \sum_{ij} \epsilon_{ij} |i,j\rangle \langle i,j' | + \sum_{n,n'} t(i,j) |i,j\rangle \langle i',j'|.
\]

on the square lattice. Here, \(\epsilon_{ij}\) is the potential at the lattice site \((i,j)\) and \(t = \hbar^2/(2ma^2)\) is the nearest-neighbor hopping amplitude with \(m\) being the effective mass of the electron. The sum in the second term is over nearest neighbors. In the presence of a magnetic field, \(t\) acquires the Peierls phase factor. We incorporate a potential disorder by assuming \(\epsilon_{ij}\) being distributed randomly within an interval \([-D/2, D/2]\). The random potential is assigned for all the lattice sites for the bulk disorder. For the boundary disorder, the random potential is restricted to the boundary lattice sites, such as those shown as the green dots in Fig. 1(b). The Coulomb interaction is taken into account by adding to the Hamiltonian an onsite repulsion term

\[
H_{int} = \sum \rho_{i,\sigma}(E),
\]

where \(n_{i,\sigma} = c_{i,\sigma}^\dagger c_{i,\sigma}\) is the number operator at the lattice site \((i,j)\) with spin \(\sigma\). \(c^\dagger\) and \(c\) are the creation and annihilation operators, respectively. The interaction term is treated within the mean-field approximation.

The average number of electrons is thereby evaluated by integrating the local density-of-states \(\rho_{i,\sigma}(E)\) over energy \(E\) up to the chemical potential \(\mu\) as

\[
\langle n_{i,\sigma}\rangle = \int_{-\infty}^{\mu} \rho_{i,\sigma}(E)dE.
\]

Green’s functions \(G^\pm(E) = (E - H \mp i\delta)^{-1}\) defined for the Hamiltonian \(H = H_0 + H_{int}\), which were used also to obtain the equilibrium transmission coefficients through the channel, were calculated using the recursion technique.

The space charge distribution was determined to satisfy the self-consistency.

For the nonequilibrium transport, the current \(I\) induced by an applied voltage \(V = (\mu_1 - \mu_2)/e\) was calculated using the nonequilibrium-Green’s-function technique.

In this formalism, the Dyson equation is written with respect to

\[
\hat{G} = \begin{bmatrix} 0 & G^- \\ G^+ & F \end{bmatrix},
\]

The local current in the \(x\) direction along the channel is given by

\[
I_{ij}^x = e \frac{t}{2\hbar} \int [F_{ij}(i+1, i) - F_{ij}(i, i+1)]d\omega.
\]

The total current \(I\) is hence obtained as

\[
I = \sum_j I_{ij}^x.
\]

The recursion formulae to calculate \(\hat{G}\) can be found in Ref. 23.

REFERENCES


27. A large portion of the computation time for Fig. 6 was used for the quantum Hall regime. The difference in the simulation time between the low- and high-magnetic-field regimes was orders of magnitude larger than that implied by the difference in $N$ in Fig. 6(c).